

Sheaves for non-categorists - part 2  
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<http://angg.twu.net/math-b.html>

Take a set of “worlds”,  $W$ , and a directed acyclical graph on  $W$ , given by a relation  $R \subset W \times W$ . Let’s call the functions  $W \rightarrow \{0, 1\}$  “modal truth-values”, and the  $R$ -non-decreasing functions  $W \rightarrow \{0, 1\}$  “intuitionistic truth-values”. If we see  $W$  as a topological space with the order topology induced by  $R$ , the intuitionistic truth-values correspond to open sets.

The pair  $(\mathcal{O}(W), \subseteq)$  is a Heyting algebra — meaning that we can interpret intuitionistic propositional logic on it — and it is a (bigger) DAG, and so we can repeat the above process with it, to generate a (bigger) topological space  $(\mathcal{O}(W), \mathcal{O}(\mathcal{O}(W)))$ , which is the natural setting for talking about “covers”, “saturated covers”, and “unions of covers”.

This presentation will be focused on understanding all these ideas (and more!), mainly in the case where  $W$  has three worlds forming a “V”, and  $R$  has two arrows pointing downwards. The operation of “taking the union of a cover” turns out to be a particular case of a “Lawvere-Tierney modality”; the double negation is another LT-modality.