Cálculo 2
PURO-UFF - 2016.1
VR - 3/ago/2016 - Eduardo Ochs
Links importantes:
http://angg.twu.net/2016.1-C2.html (página do curso)
http://angg.twu.net/2016.1-C2/2016.1-C2.pdf (quadros)
http://angg.twu.net/LATEX/2016-1-C2-VR.pdf (esta prova)
eduardoochs@gmail.com (meu e-mail)

1) **(Total: 6.0)** Calcule:

$$a) \quad (\mathbf{2.0pts}) \quad \int \frac{x^3}{x^2-4}\,dx$$

$$b) \quad (\mathbf{2.0pts}) \quad \int \frac{1}{x\sqrt{x^2-1}}\,dx$$

$$c) \quad (\mathbf{2.0pts}) \quad \int_{x=-2}^{x=2} |x^2-1|\,dx$$

2) **(Total: 2.0)** Seja (*) esta EDO: $f'' + 2f' + 5f = 0$.
a) **(1.0 pts)** Encontre as soluções básicas reais de $(*)$.
b) **(1.0 pts)** Encontre uma solução de $(*)$ que obedeça $f(0) = 0$ e $f'(0) = 1$.

3) **(Total: 1.0)** Seja (**) esta EDO: $f'(x) = (2x-2)/e^{f(x)}$.
Encontre a solução geral de (**).

4) **(Total: 1.5)**
a) **(0.5 pts)** Teste a sua solução da 1a.
a) **(0.5 pts)** Teste a sua solução da 1b.
a) **(0.5 pts)** Teste a sua solução da 3.

Método de Heaviside:
Se $f(x) = \frac{\alpha}{x-a} + \frac{\beta}{x-b} + \frac{\gamma}{x-c} = \frac{p(x)}{(x-a)(x-b)(x-c)}$,
então $\lim_{x\to a} f(x)(x-a) = \alpha = \frac{p(a)}{(a-b)(a-c)}$.

Substituição:

$$\begin{array}{l} g(h(x))|_{x=a}^{x=b} = \int_{x=a}^{x=b} g'(h(x))\frac{d\,h(x)}{dx}\,dx \\ \quad || \\ g(u))|_{u=h(a)}^{u=h(b)} = \int_{u=h(a)}^{u=h(b)} g'(u)\,du \end{array}$$

Fórmulas:

$$\begin{array}{ll} \int_{x=a}^{x=b} f(g(x))\frac{d\,g(x)}{dx}\,dx & \int f(g(x))\frac{d\,g(x)}{dx}\,dx \\ = \int_{x=a}^{x=b} f(u)\frac{du}{dx}\,dx & = \int f(u)\frac{du}{dx}\,dx \quad \left[\,u=g(x)\,\right] \\ = \int_{u=g(a)}^{u=g(b)} f(u)\,du & = \int f(u)\,du \quad \left[\,u=g(x)\,\right] \end{array}$$

Substituição inversa:

$$\begin{array}{l} g(h(x))|_{x=h^{-1}(\alpha)}^{x=h^{-1}(\beta)} = \int_{x=h^{-1}(\alpha)}^{x=h^{-1}(\beta)} g'(h(x))\frac{d\,h(x)}{dx}\,dx \\ \quad || \\ g(u))|_{u=h(h^{-1}(\alpha))}^{u=h(h^{-1}(\beta))} = \int_{u=h(h^{-1}(\alpha))}^{u=h(h^{-1}(\beta))} g'(u)\,du \\ \quad || \\ g(u))|_{u=\alpha}^{u=\beta} = \int_{u=\alpha}^{u=\beta} g'(u)\,du \end{array}$$

Fórmulas:

$$\begin{array}{ll} \int_{u=\alpha}^{u=\beta} f(u)\,du & \int f(u)\,du \\ = \int_{x=g^{-1}(\alpha)}^{x=g^{-1}(\beta)} f(u)\frac{du}{dx}\,dx & = \int f(u)\frac{du}{dx}\,dx \quad \left[\begin{smallmatrix} u=g(x) \\ x=g^{-1}(u) \end{smallmatrix}\right] \\ = \int_{x=g^{-1}(\alpha)}^{x=g^{-1}(\beta)} f(g(x))\frac{d\,g(x)}{dx}\,dx & = \int f(g(x))\frac{d\,g(x)}{dx}\,dx \quad \left[\,x=g^{-1}(u)\,\right] \end{array}$$

Substituição trigonométrica:

$$\begin{array}{ll} \int_{s=a}^{s=b} F(s,\sqrt{1-s^2})\,ds & \int F(s,\sqrt{1-s^2})\,ds \\ = \int_{\theta=\text{arcsen}\,a}^{\theta=\text{arcsen}\,b} F(\text{sen}\,\theta,\sqrt{1-\text{sen}^2\theta})\frac{d\,\text{sen}\,\theta}{d\theta}\,d\theta & = \int F(s,\sqrt{1-s^2})\frac{ds}{d\theta}\,d\theta \quad \left[\begin{smallmatrix} s=\text{sen}\,\theta \\ \theta=\text{arcsen}\,\theta \end{smallmatrix}\right] \\ = \int_{\theta=\text{arcsen}\,a}^{\theta=\text{arcsen}\,b} F(\text{sen}\,\theta,\cos\theta)\cos\theta\,d\theta & = \int F(s,c)c\,d\theta \quad \left[\begin{smallmatrix} s=\text{sen}\,\theta \\ c=\cos\theta \\ \theta=\text{arcsen}\,\theta \end{smallmatrix}\right] \end{array}$$

$$\begin{array}{ll} \int_{z=a}^{z=b} F(z,\sqrt{z^2-1})\,dz & \int F(z,\sqrt{z^2-1})\,dz \\ = \int_{\theta=\text{arcsec}\,a}^{\theta=\text{arcsec}\,b} F(\sec\theta,\sqrt{\sec^2\theta-1})\frac{d\sec\theta}{d\theta}\,d\theta & = \int F(z,\sqrt{z^2-1})\frac{dz}{d\theta}\,d\theta \quad \left[\begin{smallmatrix} z=\sec\theta \\ \theta=\text{arcsec}\,z \end{smallmatrix}\right] \\ = \int_{\theta=\text{arcsec}\,a}^{\theta=\text{arcsec}\,b} F(\sec\theta,\tan\theta)\sec\theta\tan\theta\,d\theta & = \int F(z,t)zt\,d\theta \quad \left[\begin{smallmatrix} z=\sec\theta \\ \theta=\text{arcsec}\,z \\ t=\tan\theta \end{smallmatrix}\right] \end{array}$$

$$\begin{array}{ll} \int_{t=a}^{t=b} F(t,\sqrt{1+t^2})\,dt & \int F(t,\sqrt{1+t^2})\,dt \\ = \int_{\theta=\arctan a}^{\theta=\arctan b} F(\tan\theta,\sqrt{1+\tan^2\theta})\frac{d\tan\theta}{d\theta}\,d\theta & = \int F(t,\sqrt{1+t^2})\frac{dt}{d\theta}\,d\theta \quad \left[\begin{smallmatrix} t=\tan\theta \\ \theta=\arctan t \end{smallmatrix}\right] \\ = \int_{\theta=\arctan a}^{\theta=\arctan b} F(\tan\theta,\sec\theta)\sec^2\theta\,d\theta & = \int F(t,z)z^2\,d\theta \quad \left[\begin{smallmatrix} t=\tan\theta \\ \theta=\arctan t \\ z=\sec\theta \end{smallmatrix}\right] \end{array}$$

Mini-gabarito:
(não revisado, contém erros)

1a) $\frac{x^3}{x^2-4} = x + \frac{2}{x+2} + \frac{2}{x-2}$
$\int \frac{x^3}{x^2-4}\,dx = \int x + \frac{2}{x+2} + \frac{2}{x-2}\,dx = \frac{x^2}{2} + 2\ln|x+2| + 2\ln|x-2|$

1b) $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x$

1c) $\int_{x=-2}^{x=2} |x^2-1|\,dx = \int_{x=-2}^{x=-1}(x^2-1)\,dx + \int_{x=-1}^{x=1}(1-x^2)\,dx + \int_{x=1}^{x=2}(x^2-1)\,dx$
$= \frac{4}{3} + \frac{4}{3} + \frac{4}{3} = 4$

2a) $f_1(x) = e^{-x}\cos 2x,\ f_2(x) = e^{-x}\operatorname{sen} 2x.$
2b) $f_3(x) = \frac{1}{2}e^{-x}\operatorname{sen} 2x.$

3) $\frac{dy}{dx} = \frac{2x-2}{e^y}$
$e^{-y}dy = (2x-2)dx$
$\int e^{-y}\,dy = \int 2x-2\,dx + C_1$
$-e^{-y} = x^2 - 2x + C_1$
$e^{-y} = 2x - x^2 + C_2$
$-y = \ln(2x - x^2 + C_2)$
$y = -\ln(2x - x^2 + C_2)$