Geometria Analítica
PURO-UFF - 2016.1
P1 - 8/jun/2016 - Eduardo Ochs
Respostas sem justificativas não serão aceitas.
Proibido usar quaisquer aparelhos eletrônicos.

1) **(Total: 2.5)** Sejam

$$\begin{aligned} r_1 &= \{\,(x,y)\in\mathbb{R}^2 \mid 5x-2y=0\,\},\\ r_2 &= \{\,(2,2)+t\overrightarrow{(2,-1)} \mid t\in\mathbb{R}\,\},\\ r_3 &= \{\,(5+3u,1-4u) \mid u\in\mathbb{R}\,\}.\end{aligned}$$

a) **(1.5 pts)** Determine as coordenadas de $C\in r_1\cap r_2$, $D\in r_1\cap r_3$ e $E\in r_2\cap r_3$. Obs: se você conseguir encontrar as coordenadas de algum ponto só pelo gráfico basta provar que ele pertence às retas adequadas.
b) **(1.0 pts)** Determine a área do triângulo CDE.

2) **(Total: 2.5)** Sejam:

$$C=\{\,(x,y)\in\mathbb{R}^2 \mid (x-3)^2+(y-4)^2=25\,\}$$
$$C'=\{\,(x,y)\in\mathbb{R}^2 \mid (x-7)^2+(y-3)^2=4\,\}$$
$$C\cap C'=\{I,I'\}$$

a) **(2.0 pts)** Encontre os dois pontos de interseção I e I' dos dois círculos. Obs: se você conseguir encontrar algum ponto só pelo gráfico basta provar que ele pertence aos dois círculos.
b) **(0.5 pts)** Determine o ponto médio M de I e I'.

3) **(Total: 1.5)** Verdadeiro ou falso? Justifique.
Se $\vec{u}$ e $\vec{v}$ são ortogonais e não-nulos e $\vec{w}=a\vec{u}+b\vec{v}$ então $\vec{w}=\mathrm{Pr}_{\vec{u}}\vec{w}+\mathrm{Pr}_{\vec{v}}\vec{w}$.

4) **(Total: 1.0)** Determine a distância entre as retas com equações $y=1-\frac{x}{3}$ e $y=2-\frac{x}{3}$.

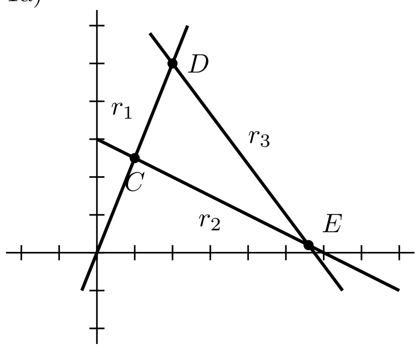
5) **(Total: 2.5)** Sejam $A=(2,5)$, $B=(1,3)$, $C=(2,3)$, $D=(2,1)$,

$$r=\{\,(x,y)\in\mathbb{R}^2 \mid x+y=3\,\}.$$

a) **(0.5 pts)** Calcule $\cos(A\hat{B}C)$.
b) **(2.0 pts)** Encontre uma reta s que passa por D e que faz com r o mesmo ângulo que $A\hat{B}C$.

Mini-gabarito:

1a)



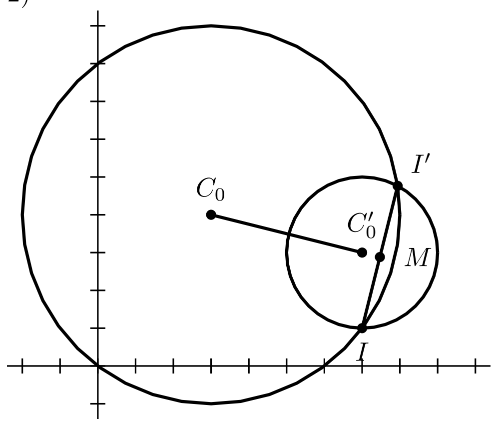
Se $C = (1, 2.5)$ então
$C \in r_1$ porque $5 \cdot 1 - 2 \cdot 2.5 = 0$,
$C \in r_2$ porque $C = (2,2) + (-0.5)\overrightarrow{(2,-1)}$,
portanto $C \in r_1 \cap r_2$.

Se $D = (2, 5)$ então
$D \in r_1$ porque $5 \cdot 2 - 2 \cdot 5 = 0$,
$D \in r_3$ porque $D = (5 + 3 \cdot 0, 1 - 4 \cdot 0)$,
portanto $C \in r_1 \cap r_3$.

Se $E = (x, y) \in r_2 \in r_3$ então
$E = (2 + 2t, 2 - t) = (5 + 3u, 1 - 4u)$
$2 + 2t = 5 + 3u$
$2 - t = 1 - 4u$
$2 - 1 + 4u = t$
$t = 1 + 4u$
$2 + 2(1 + 4u) = 5 + 3u$
$2 + 2 - 5 + 8u - 3u = 0$
$5u = 1$
$u = \frac{1}{5}$
$t = 1 + 4\frac{1}{5} = \frac{9}{5}$
$E = (x, y) = (2 + 2\frac{9}{5}, 2 - \frac{9}{5}) = (5 + 3\frac{1}{5}, 1 - 4\frac{1}{5}) = (\frac{28}{5}, \frac{1}{5}) = (5.6, 0.2)$

1b) $\overrightarrow{CD} = \overrightarrow{(1, 2.5)}$, $\overrightarrow{CE} = \overrightarrow{(4.6, -2.3)}$, $[\overrightarrow{CD}, \overrightarrow{CE}] = \begin{pmatrix} 1 & 2.5 \\ 4.6 & -2.3 \end{pmatrix}$
$|[\overrightarrow{CD}, \overrightarrow{CE}]| = \begin{vmatrix} 1 & 2.5 \\ 4.6 & -2.3 \end{vmatrix} = -2.3 - 4.6 \cdot 2.5 = -2.3 - 11.5 = -13.8$
Area$(CDE) = 13.8/2 = 6.9$

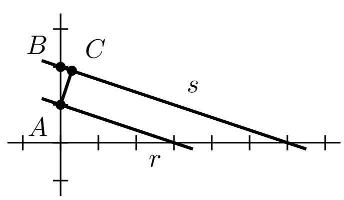
2)



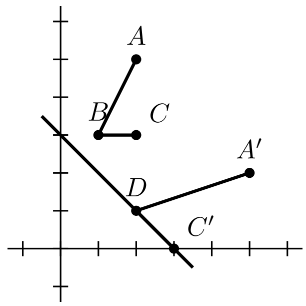
Seja $I=(7,1)$. Então
$I \in C$ porque $(7-3)^2+(1-4)^2=25$ e
$I \in C'$ porque $(7-7)^2+(1-3)^2=4$.
Sejam $\vec{u}=\overrightarrow{C_0C_0'}$, $\vec{v}=\overrightarrow{C_0I}$, $\vec{w}=\mathrm{Pr}_{\vec{u}}\vec{v}$, $M=C_0+\vec{w}$, $I'=M+\overrightarrow{IM}$.
Então $\vec{u}=\overrightarrow{(4,-1)}$, $\vec{v}=\overrightarrow{(4,-3)}$,
$\vec{w}=\frac{\overrightarrow{(4,-1)}\cdot\overrightarrow{(4,-3)}}{\overrightarrow{(4,-1)}\cdot\overrightarrow{(4,-1)}}\overrightarrow{(4,-1)}=\frac{19}{17}\overrightarrow{(4,-1)}=\overrightarrow{(\frac{76}{17},-\frac{19}{17})}$,
$M=(3,4)+\overrightarrow{(\frac{76}{17},-\frac{19}{17})}=(\frac{51}{17},\frac{68}{17})+\overrightarrow{(\frac{76}{17},-\frac{19}{17})}=(\frac{127}{17},\frac{49}{17})$
$\overrightarrow{IM}=(\frac{127}{17},\frac{49}{17})-(7,1)=(\frac{127}{17},\frac{49}{17})-(\frac{119}{17},\frac{17}{17})=\overrightarrow{(\frac{8}{17},\frac{32}{17})}$
$I'=(\frac{127}{17},\frac{49}{17})+\overrightarrow{(\frac{8}{17},\frac{32}{17})}=(\frac{135}{17},\frac{81}{17})$

3) $\mathrm{Pr}_{\vec{u}}\vec{w}=\mathrm{Pr}_{\vec{u}}(a\vec{u}+b\vec{v})=$
$\frac{\vec{u}\cdot(a\vec{u}+b\vec{v})}{\vec{u}\cdot\vec{u}}\vec{u}=\frac{\vec{u}\cdot(a\vec{u})+\vec{u}\cdot(b\vec{v})}{\vec{u}\cdot\vec{u}}\vec{u}=\frac{a(\vec{u}\cdot\vec{u})+b(\vec{u}\cdot\vec{v})}{\vec{u}\cdot\vec{u}}\vec{u}=\frac{a(\vec{u}\cdot\vec{u})+b\cdot 0}{\vec{u}\cdot\vec{u}}\vec{u}=\frac{a(\vec{u}\cdot\vec{u})}{\vec{u}\cdot\vec{u}}\vec{u}=a\vec{u}$
$\mathrm{Pr}_{\vec{v}}\vec{w}=\mathrm{Pr}_{\vec{v}}(a\vec{u}+b\vec{v})=$
$\frac{\vec{v}\cdot(a\vec{u}+b\vec{v})}{\vec{v}\cdot\vec{v}}\vec{v}=\frac{\vec{v}\cdot(a\vec{u})+\vec{v}\cdot(b\vec{v})}{\vec{v}\cdot\vec{v}}\vec{v}=\frac{a(\vec{v}\cdot\vec{u})+b(\vec{v}\cdot\vec{v})}{\vec{v}\cdot\vec{v}}\vec{v}=\frac{a\cdot 0+b(\vec{v}\cdot\vec{v})}{\vec{v}\cdot\vec{v}}\vec{v}=\frac{b(\vec{v}\cdot\vec{v})}{\vec{v}\cdot\vec{v}}\vec{v}=b\vec{v}$
$\mathrm{Pr}_{\vec{u}}\vec{w}+\mathrm{Pr}_{\vec{v}}\vec{w}=a\vec{u}+b\vec{v}=\vec{w}$

4)



Sejam:
$r=\{\,(x,y)\in\mathbb{R}^2 \mid y=1-\frac{x}{3}\,\}$, $A=(0,1)$,
$s=\{\,(x,y)\in\mathbb{R}^2 \mid y=2-\frac{x}{3}\,\}$, $B=(0,2)$.
Então $d(r,s)=d(A,s)=\frac{d(A,B)}{1+(-1/3)^2}=\frac{1}{10/9}=\frac{9}{10}$.

5) 

5a) $\cos(A\hat{B}C) = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{||\overrightarrow{BA}||\,||\overrightarrow{BC}||} = \frac{1}{\sqrt{5}}$

5b) $\overrightarrow{(1,-1)} + 2\overrightarrow{(1,1)} = \overrightarrow{(3,1)}$

$s = \{\, (2,1) + t\overrightarrow{(3,1)} \mid t \in \mathbb{R} \,\}$