Geometria Analítica
PURO-UFF - 2016.2
P2 - 18/jan/2017 - Eduardo Ochs
Respostas sem justificativas não serão aceitas.
Proibido usar quaisquer aparelhos eletrônicos.

Lembre que uma equação de cônica é uma equação da forma $ax^2 + bxy + cy^2 + dy + ey + f = 0$; $4 + (x + y)(x - y) = 5y$ não é uma equação de cônica mas é equivalente a uma: $x^2 - y^2 - 5y + 4 = 0$.
Nas questões 3 e 4 vamos usar a abreviação $[\text{equação}] = \{\, (x, y, z) \in \mathbb{R}^3 \mid \text{equação} \,\}$.

1) **(Total: 4.0)** Faça esboços das cônicas com as equações abaixo. Algumas delas são degeneradas.

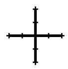
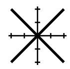
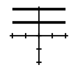
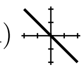
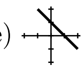
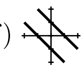
a) (0.1 pts)	$xy = 0$	i) (0.4 pts)	$(x + y)^2 + (y + 2)^2 = 0$
b) (0.1 pts)	$(x + y)(x - y) = 0$	j) (1.0 pts)	$(x + y)^2 + (y + 2)^2 = 1$
c) (0.1 pts)	$(y - 1)(y - 2) = 0$	k) (0.4 pts)	$(x + y)(y + 2) = 0$
d) (0.1 pts)	$x + y = 0$	l) (1.0 pts)	$(x + y)(y + 2) = 1$
e) (0.1 pts)	$x + y = 1$	m) (0.4 pts)	$(\frac{x-4}{2})^2 + (\frac{y-6}{3})^2 = 1$
f) (0.1 pts)	$(x + y)^2 = 1$		
g) (0.1 pts)	$y + 2 = 0$		
h) (0.1 pts)	$y + 2 = 1$		

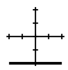
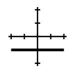
2) **(Total: 1.0)** Encontre os focos das elipses cujos pontos óbvios são:
a) **(0.5 pts)** $(\pm 3, 0)$, $(0, \pm 5)$
b) **(0.5 pts)** $(4 \pm 3, 2)$, $(4, 2 \pm 5)$

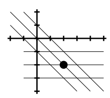
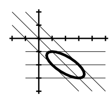
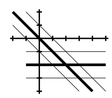
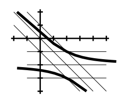
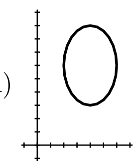
3) **(Total: 1.5)** Sejam $\pi_1 = [y = 3 - 2x]$, $\pi_2 = [x + y + z = 4]$, $r = \pi_1 \cap \pi_2$.
a) **(0.5 pts)** Encontre o ponto de r que tem $x = 0$.
b) **(0.5 pts)** Encontre o ponto de r que tem $x = 1$.
c) **(0.5 pts)** Se $r' = \{\, (x, ax + b, cx + d) \mid x \in \mathbb{R} \,\}$ e $r = r'$, quem são a, b, c e d?

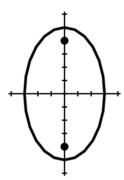
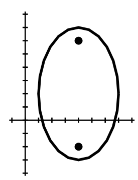
4) **(Total: 3.5)** Sejam $A = (1, 0, 0)$, $B = (0, 1, 0)$, $\vec{u} = \vec{AB}$, $C = (1, 1, 0)$, $D = (1, 0, 1)$, $\vec{v} = \vec{CD}$, $r = \{\, A + t\vec{u} \mid t \in \mathbb{R} \,\}$, $r' = \{\, C + t'\vec{v} \mid t' \in \mathbb{R} \,\}$.
a) **(1.5 pts)** Demonstre (algebricamente!) que r e r' são reversas, ou, equivalentemente, que A, B, C e D não são coplanares.
b) **(2.0 pts)** Calcule a distância entre r e r'.

Gabarito:

1a)  1b)  1c)  1d)  1e)  1f) 

1g)  1h) 

1i)  1j)  1k)  1l)  1m) 

2a)  2b) 

3a) $x=0 \ \Rightarrow\ y=3 \ \Rightarrow\ z=1 \ \Rightarrow\ P=(0,3,1)$
3b) $x=1 \ \Rightarrow\ y=1 \ \Rightarrow\ z=2 \ \Rightarrow\ P=(1,1,2)$
3c) $r' = \{\, (x, -2x+3, 1x+1) \mid x \in \mathbb{R} \,\}$, $a=-2$, $b=3$, $c=1$ e $d=1$

4a) $[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = [\overrightarrow{(-1,1,0)}, \overrightarrow{(0,1,0)}, \overrightarrow{(0,0,1)}] = \begin{vmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1 \neq 0$,
portanto A, B, C e D não são coplanares.
4b) Seja $\vec{w} = \overrightarrow{AC} = \overrightarrow{(0,1,0)}$. Então

$$\begin{aligned} d(r,r') &= [\overrightarrow{AB}, \overrightarrow{CD}, \overrightarrow{AC}]/||\overrightarrow{AB} \times \overrightarrow{CD}|| \\ &= [\overrightarrow{(-1,1,0)}, \overrightarrow{(0,-1,1)}, \overrightarrow{(0,1,0)}]/||\overrightarrow{(-1,1,0)} \times \overrightarrow{(0,-1,1)}|| \\ &= \begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{vmatrix} /||\overrightarrow{(1,1,1)}|| \\ &= 1/\sqrt{3} \\ &= \sqrt{3}/3 \end{aligned}$$