S5 is a Paraconsistent Logic

In Jean-Yves Beziau's paper
"S5 is a Paraconsistent Logic and so is first-order classical logic" (2002)
(See http://www.jyb-logic.org/papers.html)
he defines a paraconsistent negation $\sim$ on S5 as $\sim a := \Diamond\neg a$,
and points out that $a, \sim a \vdash b$ does not hold but 3, 4, 5 below are theorems:

1) $\sim a := \Diamond\neg a$
2) $a, \Diamond\neg a \not\vdash b$
3) $a \vee \sim a$
4) $(a \to \sim a) \to \sim a$
5) $(\sim a \to a) \to a$

He also points out that the implications 6-15 below (middle column),
are strict in S5 with $\sim a := \Diamond\neg a$ (right column);
note that they are all biimplications clasically (left column).

6)	$(a \to b) \leftrightarrow (\neg a \vee b)$	$(a \to b) \to (\sim a \vee b)$	$(a \to b) \not\leftarrow (\sim a \vee b)$
7)	$\neg\neg a \leftrightarrow a$	$\sim\sim a \to a$	$\sim\sim a \not\leftarrow a$
8)	$\neg(a \wedge b) \leftrightarrow (\neg a \vee \neg b)$	$\sim (a \wedge b) \to (\sim a \vee \sim b)$	$\sim (a \wedge b) \not\leftarrow (\sim a \vee \sim b)$
9)	$\neg(\neg a \wedge \neg b) \leftrightarrow (a \vee b)$	$\sim (\sim a \wedge \sim b) \to (a \vee b)$	$\sim (\sim a \wedge \sim b) \not\leftarrow (a \vee b)$
10)	$\neg(a \wedge \neg b) \leftrightarrow (\neg a \vee b)$	$\sim (a \wedge \sim b) \to (\sim a \vee b)$	$\sim (a \wedge \sim b) \not\leftarrow (\sim a \vee b)$
11)	$\neg(\neg a \wedge b) \leftrightarrow (a \vee \neg b)$	$\sim (\sim a \wedge b) \to (a \vee \sim b)$	$\sim (\sim a \wedge b) \not\leftarrow (a \vee \sim b)$
12)	$\neg(\neg a \vee b) \leftrightarrow (a \wedge \neg b)$	$\sim (\sim a \vee b) \to (a \wedge \sim b)$	$\sim (\sim a \vee b) \not\leftarrow (a \wedge \sim b)$
13)	$\neg(a \vee b) \leftrightarrow (\neg a \wedge \neg b)$	$\sim (a \vee b) \to (\sim a \wedge \sim b)$	$\sim (a \vee b) \not\leftarrow (\sim a \wedge \sim b)$
14)	$\neg(a \vee \neg b) \leftrightarrow (\neg a \wedge b)$	$\sim (a \vee \sim b) \to (\sim a \wedge b)$	$\sim (a \vee \sim b) \not\leftarrow (\sim a \wedge b)$
15)	$\neg(\neg a \vee \neg b) \leftrightarrow (a \wedge b)$	$\sim (\sim a \vee \sim b) \to (a \wedge b)$	$\sim (\sim a \vee \sim b) \not\leftarrow (a \wedge b)$

It easy to convince "children" of the sentences with '$\not\leftarrow$' in the "strictness" column — we just need to show a model $M = (W, R, v)$ in which the corresponding sentence with '$\leftarrow$' is not '$\top$'. For example:

6) $$\underbrace{(\underbrace{\underbrace{a}_{001} \to \underbrace{b}_{000}}_{110}) \leftarrow (\underbrace{\underbrace{\sim \underbrace{a}_{001}}_{111} \vee \underbrace{b}_{000}}_{111})}_{110}$$

7) $$\underbrace{\underbrace{\sim \underbrace{\sim \underbrace{a}_{001}}_{111}}_{000} \leftarrow \underbrace{a}_{001}}_{110}$$

Note that in the models above, and in the ones used in the next page,
we have $W = \{1, 2, 3\}$ and $R = W \times W$;
(W, R) is the transitive-reflexive closure of the directed graph $1 \leftrightarrow 2 \leftrightarrow 3$.
The positional notations we use here are explained in detail in
sections 1, 2, 9, 12 of http://angg.twu.net/LATEX/2016planar-has.pdf
(but here we are omitting *horizontal* arrows).

8) $$\underbrace{\underbrace{\sim\underbrace{(\underbrace{a}\wedge\underbrace{b})}}\leftarrow\underbrace{(\underbrace{\sim\underbrace{a}}\vee\underbrace{\sim\underbrace{b}})}}$$

9) $$\underbrace{\underbrace{\sim(\underbrace{\underbrace{\sim\underbrace{a}_{001}}_{110}\wedge\underbrace{\sim\underbrace{b}_{010}}_{111}}_{111})}_{000}\leftarrow(\underbrace{\underbrace{a}_{001}\vee\underbrace{b}_{010}}_{011})}_{100}$$

10) $$\underbrace{\underbrace{\sim(\underbrace{\underbrace{a}_{111}\wedge\underbrace{\sim\underbrace{b}_{001}}_{111}}_{111})}_{000}\leftarrow(\underbrace{\underbrace{\sim\underbrace{a}_{111}}_{000}\vee\underbrace{b}_{001}}_{001})}_{110}$$

11) $$\underbrace{\underbrace{\sim(\underbrace{\underbrace{\sim\underbrace{a}_{001}}_{111}\wedge\underbrace{b}_{111}}_{111})}_{000}\leftarrow(\underbrace{\underbrace{a}_{001}\vee\underbrace{\sim\underbrace{b}_{111}}_{000}}_{001})}_{110}$$

12) $$\underbrace{\underbrace{\sim(\underbrace{\underbrace{\sim\underbrace{a}_{001}}_{111}\vee\underbrace{b}_{010}}_{111})}_{000}\leftarrow(\underbrace{\underbrace{a}_{001}\wedge\underbrace{\sim\underbrace{b}_{010}}_{111}}_{001})}_{110}$$

13) $$\underbrace{\underbrace{\sim(\underbrace{\underbrace{a}_{011}\vee\underbrace{b}_{110}}_{111})}\leftarrow(\underbrace{\underbrace{\sim\underbrace{a}_{011}}_{111}\wedge\underbrace{\sim\underbrace{b}_{110}}_{111}}_{111})}_{000}$$

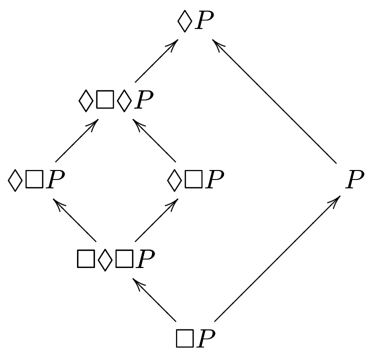
14) $$\underbrace{\underbrace{\sim(\underbrace{\underbrace{a}_{010}\vee\underbrace{\sim\underbrace{b}_{001}}_{111}}_{111})}_{000}\leftarrow(\underbrace{\underbrace{\sim\underbrace{a}_{010}}_{111}\wedge\underbrace{b}_{001}}_{001})}_{110}$$

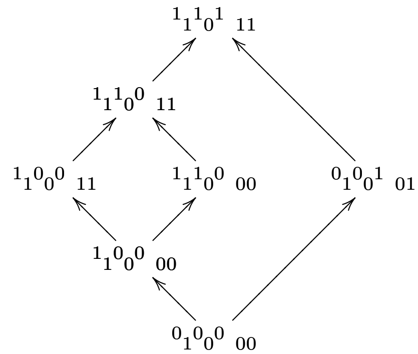
15) $$\underbrace{\underbrace{\sim(\underbrace{\underbrace{\sim\underbrace{a}_{011}}_{111}\vee\underbrace{\sim\underbrace{b}_{110}}_{111}}_{111})}_{000}\leftarrow(\underbrace{\underbrace{a}_{011}\wedge\underbrace{b}_{110}}_{010})}_{101}$$

Modalities in S4

(See Chellas, p.149)

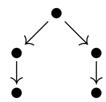
https://groups.google.com/a/dimap.ufrn.br/forum/#!msg/logica-l/JxptOElKnIO/h-ZX6qmvDwAJ





P	$=$	${}^{0}{}_{1}{}^{0}{}_{0}{}^{1}\;{}_{01}$	P	$=$	${}^{0}{}_{1}{}^{0}{}_{0}{}^{1}\;{}_{01}$
$\Box P$	$=$	${}^{0}{}_{1}{}^{0}{}_{0}{}^{0}\;{}_{00}$	$\Diamond P$	$=$	${}^{1}{}_{1}{}^{1}{}_{0}{}^{1}\;{}_{11}$
$\Diamond\Box P$	$=$	${}^{1}{}_{1}{}^{1}{}_{0}{}^{0}\;{}_{00}$	$\Box\Diamond P$	$=$	${}^{1}{}_{1}{}^{0}{}_{0}{}^{0}\;{}_{11}$
$\Box\Diamond\Box P$	$=$	${}^{1}{}_{1}{}^{0}{}_{0}{}^{0}\;{}_{00}$	$\Diamond\Box\Diamond P$	$=$	${}^{1}{}_{1}{}^{1}{}_{0}{}^{0}\;{}_{11}$

Intuitionistic logic with open sets as truth values

$(W,R) =$ 

$$\underbrace{\underbrace{\neg(\underbrace{\underbrace{P}_{\substack{0\\0\,0\\1\,0}} \land \underbrace{Q}_{\substack{0\\0\,0\\0\,1}}}_{\substack{0\\0\,0\\0\,0}})}_{\substack{1\\1\,1\\1\,1}} \to \underbrace{(\underbrace{\neg\underbrace{P}_{\substack{0\\0\,0\\1\,0}}}_{\substack{0\\0\,1\\0\,1}} \lor \underbrace{\neg\underbrace{Q}_{\substack{0\\0\,0\\0\,1}}}_{\substack{0\\1\,0\\1\,0}})}_{\substack{0\\1\,1\\1\,1}}}_{\substack{0\\1\,1\\1\,1}}$$

(This will probably be moved to "Planar Heyting Algebras for Children" at some point — with explanations...)