Geometria Analítica
PURO-UFF - 2017.2
P2 - 11/dez/2017 - Eduardo Ochs
Respostas sem justificativas não serão aceitas.
Diagramas muito ambíguos *serão* interpretados errado.
Proibido usar quaisquer aparelhos eletrônicos.

Lembre que uma equação de cônica é uma equação da forma $ax^2 + bxy + cy^2 + dx + ey + f = 0$; $4 + (x + y)(x - y) = 5y$ não é uma equação de cônica mas é equivalente a uma: $x^2 - y^2 - 5y + 4 = 0$. E o truque pra gente se livrar das duas raízes quadradas em $\sqrt{A} + \sqrt{B} = C$ ou $\sqrt{A} - \sqrt{B} = C$ é:

$$\begin{array}{lcl} \sqrt{A} + \sqrt{B} = C & \Rightarrow & C^2(C^2 - 2(A + B)) + (A - B)^2 = 0 \\ \sqrt{A} - \sqrt{B} = C & \Rightarrow & C^2(C^2 - 2(A + B)) + (A - B)^2 = 0 \end{array}$$

1) **(Total: 1.5)** Use as fórmulas acima para transformar

$$d((x, y), (-2, 0)) - d((x, y), (2, 0)) = 2$$

numa equação de cônica e verifique se os seguintes pontos obedecem a equação original e a equação da cônica: $(1, 0)$, $(2, 3)$, $(-2, 3)$, $(-1, 0)$.

2) **(Total: 4.5)** Faça esboços das cônicas com as equações abaixo. Algumas delas são degeneradas. Em todos os itens abaixo considere que $u = 2y$ e $v = x+y$ — ou que u e v são *abreviações* para $2y$ e $x + y$.

a)	(0.1 pts)	$x^2 + y^2 = 1$	k)	(0.5 pts)	$u(u - 1) = 0$
b)	(0.1 pts)	$x^2 + y^2 = 4$	l)	(0.5 pts)	$v(v - 1) = 0$
c)	(0.1 pts)	$x^2 + y^2 = 0$	m)	(0.2 pts)	$u^2 + v^2 = 0$
d)	(0.1 pts)	$xy = 1$	n)	(0.3 pts)	$u^2 + v^2 = 1$
e)	(0.1 pts)	$xy = 4$	o)	(0.2 pts)	$uv = 0$
f)	(0.1 pts)	$xy = 0$	p)	(0.4 pts)	$uv = 1$
g)	(0.1 pts)	$xy = -1$	q)	(0.4 pts)	$uv = -1$
h)	(0.1 pts)	$x^2 = y$	r)	(0.5 pts)	$u^2 = v$
i)	(0.1 pts)	$x = y^2$	s)	(0.5 pts)	$u = v^2$
j)	(0.1 pts)	$x^2 = y^2$			

3) **(Total: 1.5)** Dê uma parametrização para a reta que pertence aos planos $x + y + z = 3$ e $x + 2z = 3$.

4) **(Total: 2.5)** Sejam:

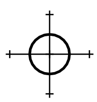
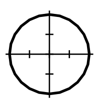
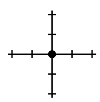
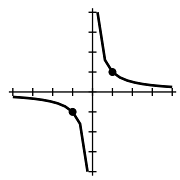
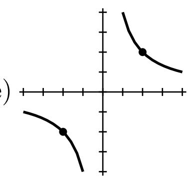
$$\begin{array}{rcl} r & = & \{\, (t, t, 0) \mid t \in \mathbb{R} \,\} \\ s & = & \{\, (2t, -2t, 4) \mid t \in \mathbb{R} \,\} \\ r' & = & \{\, (t, t, 0) \mid t \in \mathbb{R} \,\} \\ s' & = & \{\, (t, 0, 3 - t \mid t \in \mathbb{R} \,\} \end{array}$$

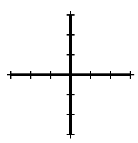
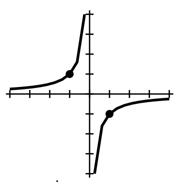
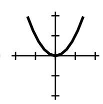
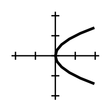
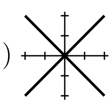
a) **(0.1 pts)** Calcule $d(r, s)$ no olhômetro.
a) **(0.9 pts)** Calcule $d(r, s)$ pela fórmula.
c) **(1.5 pts)** Calcule $d(r', s')$ pela fórmula.

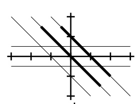
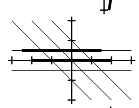
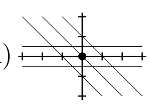
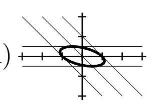
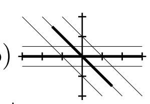
Gabarito (versão não revisada)

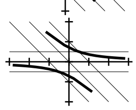
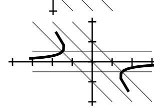
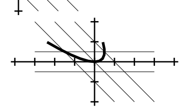
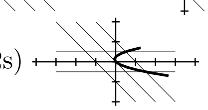
1) $d((x,y),(-2,0)) - d((x,y),(2,0)) = 2$
$\Rightarrow \sqrt{(x+2)^2+y^2} - \sqrt{(x-2)^2+y^2} = 2$. Sejam $C = 2$,
$A = (x+2)^2 + y^2 = x^2 + 4x + 4 + y^2$,
$B = (x-2)^2 + y^2 = x^2 - 4x + 4 + y^2$; então $A - B = 8x$ e
$A + B = 2(x^2 + 4 + y^2)$.
$\sqrt{A} - \sqrt{B} = C \Rightarrow C^2(C^2 - 2(A+B)) + (A-B)^2 = 0$
$\Rightarrow 4(4 - 4(x^2+4+y^2)) + (8x)^2 = 0$
$\Rightarrow (-16x^2 - 16y^2 - 64 + 16) + 64x^2 = 0$
$\Rightarrow 48x^2 - 16y^2 - 48 = 0$

(x,y)	$d((x,y),(-2,0)) - d((x,y),(2,0)) = 0$	$48x^2 - 16y^2 - 48 = 0$
$(1,0)$	$3-1=2$	$48-0-48=0$
$(2,3)$	$5-3=2$	$192-144-48=0$
$(-2,3)$	$5-3=2$	$192-144-48=0$
$(-1,0)$	$1-2\neq -2$	$48-0-48=0$

2a)  2b)  2c)  2d)  2e) 

2f)  2g)  2h)  2i)  2j) 

2k)  2l)  2m)  2n)  2o) 

2p)  2q)  2r)  2s) 

3) $z = \frac{3-x}{2}$, $y = 3 - x - 2 = 3 - x - \frac{3-x}{2} = \frac{3-x}{2}$, $r = \{\, (x, \frac{3-x}{2} \frac{3-x}{2}) \mid x \in \mathbb{R} \,\}$

4) Sejam:

$r = \{\, (0,0,0) + \overrightarrow{(1,1,0)} \mid t \in \mathbb{R} \,\}$, $\quad A = (0,0,0)$, $\quad \vec{u} = \overrightarrow{(1,1,0)}$,
$s = \{\, (0,0,4) + \overrightarrow{(2,-2,0)} \mid t \in \mathbb{R} \,\}$, $\quad B = (0,0,4)$, $\quad \vec{v} = \overrightarrow{(2,-2,0)}$,
$r' = \{\, (0,0,0) + \overrightarrow{(1,1,0)} \mid t \in \mathbb{R} \,\}$, $\quad A' = (0,0,0)$, $\quad \vec{u}' = \overrightarrow{(1,1,0)}$,
$s' = \{\, (0,0,3) + \overrightarrow{(1,0,-1)} \mid t \in \mathbb{R} \,\}$, $\quad B' = (0,0,3)$, $\quad \vec{v}' = \overrightarrow{(1,0,-1)}$,

4a) 4

4b) $d(r,s) = |([\vec{u},\vec{v},\overrightarrow{AB}]/\mathsf{área}(\vec{u},\vec{v}))| = |(16/||\overrightarrow{(0,0,4)}||)| = 4$

4c) $d(r',s') = |([\vec{u}',\vec{v}',\overrightarrow{A'B'}]/\mathsf{área}(\vec{u}',\vec{v}'))| = |(\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 3 \end{vmatrix}/||\overrightarrow{(1,1,-1)}||)| = |3/\sqrt{3}| = \sqrt{3}$