

This paper is the first in a series of three, and one way to explain the goal of the series is this. Toposes of the form $\mathbf{Set}^{(P,A^*)}$, where (P, A^*) is the preorder category associated to a 2-column graph (P, A) , are interesting to students of Topos Theory because their objects are easy to draw explicitly, and by working on them one can develop a lot of visual intuition about what certain categorical constructions “mean”; in particular, the “logic” of a $\mathbf{Set}^{(P,A^*)}$, i.e., its $\text{Sub}(1)$, is a Planar Heyting Algebra, and some ways to visualize sheaves on a $\mathbf{Set}^{(P,A^*)}$ are presented in the second paper in this series.

So: how much Topos Theory can one learn by using toposes of the form $\mathbf{Set}^{(P,A^*)}$ as the archetypal examples of toposes, and where is this person’s intuition going to fail when he or she passes to arbitrary toposes? This first paper does not discuss toposes, or even categories, but it has a small example of intuition failing: a person with very good visual thinking may be led to believe (wrongly) that the sentence S of sec.(?) is an intuitionistic theorem because it is true in all ZHAs.

A better way to explain the goal of this series of papers needs the term “children”. Many years ago, when I started learning Category Theory and Topos Theory from [McL] and [J77], I felt that I was taking far more time than reasonable to supply the diagrams that were “omitted” from the text, and treated as they were trivial exercises (see [Kro], p.(?)); I kept saying to my colleagues “I need a version ‘for children’ of this!” — but at that time this was a half-joke and it didn’t have a precise meaning. Years later it became clear that once we have a precise meaning for “children” — as people with certain style of thinking; compare with “people who think algebraically” and “people who think geometrically” — this yields *guidelines* on how to *complement* a standard categorical text, and produce auxiliary material that makes the original presentation — abstract, “for adults” — more accessible.

It turned out that the following *definition* of “children” is especially fruitful. “Children”

- 1) have trouble with very abstract definitions,
- 2) prefer to start from particular cases (and only then generalize),
- 3) handle diagrams better than algebraic notations,
- 4) like finite objects that can be drawn explicitly, and that are built just from numbers by forming lists and sets,
- 5) develop intuition about mathematical objects mostly by “playing” with them, and by learning how to do calculations quickly; *calculating* for them is much more basic than *proving theorems*,
- 6) tend to remember categorical definitions and proofs “positionally” by diagrams that are always drawn in a certain way.

Item 6 has an important consequence. Once we establish that, for example, our geometric morphisms will be drawn like this,

(*diagram*)

then all our particular cases of geometric morphisms became diagrams with the same shape of that one — [slides] and [PH3] have a medium-sized example —

and we can *transfer knowledge* from one side to the other and vice-versa; it is easy to transfer a construction done on the diagram for the general case (“for adults”) to the particular case (“for children”), and some times it is also possible to start