Cálculo 2
PURO-UFF - 2019.2
P1 - turma pequena - 30/outubro/2019 - Eduardo Ochs
Respostas sem justificativas não serão aceitas.
Proibido usar quaisquer aparelhos eletrônicos.

1) **(Total: 2.5)** Calcule

$$\int (2x+3)\sqrt{4x+5}\,dx.$$

2) **(Total: 2.5)** Calcule

$$\int x^3\sqrt{1-x^2}\,dx.$$

3) **(Total: 2.5)** Calcule

$$\int (\operatorname{sen} 5x)^2(\cos 6x)^2\,dx.$$

4) **(Total: 2.5)** Calcule

$$\int \frac{x^3}{x^2+9x+20}\,dx.$$

Algumas definições, fórmulas e substituições:

$c = \cos\theta$	$c^2 + s^2 = 1$	$\frac{ds}{d\theta} = c$	$E = c + is$
$s = \operatorname{sen}\theta$	$z^2 = t^2 + 1$	$\frac{dc}{d\theta} = -s$	$c = \frac{E+E^{-1}}{2}$
$t = \tan\theta$	$\sqrt{1-s^2} = c$	$\frac{dt}{d\theta} = z^2$	$s = \frac{E-E^{-1}}{2i}$
$z = \sec\theta$	$\sqrt{t^2+1} = z$	$\frac{dz}{d\theta} = zt$	$e^{ik\theta} + e^{-ik\theta} = 2\cos k\theta$
$E = e^{i\theta}$	$\sqrt{z^2-1} = t$		$e^{ik\theta} - e^{-ik\theta} = 2i\operatorname{sen} k\theta$

Cálculo 2
PURO-UFF - 2019.1
P1 - turma grande - 31/outubro/2019 - Eduardo Ochs
Respostas sem justificativas não serão aceitas.
Proibido usar quaisquer aparelhos eletrônicos.

1) **(Total: 2.5)** Calcule

$$\int (2x+3)\sqrt{4x+5}\,dx.$$

2) **(Total: 2.5)** Calcule

$$\int x^3\sqrt{1-x^2}\,dx.$$

3) **(Total: 2.5)** Calcule

$$\int (\operatorname{sen} 5x)^2(\cos 6x)^2\,dx.$$

4) **(Total: 2.5)** Calcule

$$\int \frac{x^3}{x^2+9x+20}\,dx.$$

Algumas definições, fórmulas e substituições:

$c = \cos\theta$	$c^2 + s^2 = 1$	$\frac{ds}{d\theta} = c$	$E = c + is$
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$t = \tan\theta$	$\sqrt{1-s^2} = c$	$\frac{dt}{d\theta} = z^2$	$s = \frac{E-E^{-1}}{2i}$
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$E = e^{i\theta}$	$\sqrt{z^2-1} = t$		$e^{ik\theta} - e^{-ik\theta} = 2i\operatorname{sen} k\theta$

Cálculo 2
PURO-UFF - 2019.1
P1 - versão pra Thais Knupp - 6/novembro/2019 - Eduardo Ochs
Respostas sem justificativas não serão aceitas.
Proibido usar quaisquer aparelhos eletrônicos.

1) **(Total: 2.5)** Calcule

$$\int (2+3x)\sqrt{4+5x}\,dx.$$

2) **(Total: 2.5)** Calcule

$$\int x^3\sqrt{1-x^2}\,dx.$$

3) **(Total: 2.5)** Calcule

$$\int (\operatorname{sen} 4x)^2(\cos 5x)^2\,dx.$$

4) **(Total: 2.5)** Calcule

$$\int \frac{x^3}{x^2+7x+12}\,dx.$$

Algumas definições, fórmulas e substituições:

$c = \cos\theta$	$c^2 + s^2 = 1$	$\frac{ds}{d\theta} = c$	$E = c + is$
$s = \operatorname{sen}\theta$	$z^2 = t^2 + 1$	$\frac{dc}{d\theta} = -s$	$c = \frac{E+E^{-1}}{2}$
$t = \tan\theta$	$\sqrt{1-s^2} = c$	$\frac{dt}{d\theta} = z^2$	$s = \frac{E-E^{-1}}{2i}$
$z = \sec\theta$	$\sqrt{t^2+1} = z$	$\frac{dz}{d\theta} = zt$	$e^{ik\theta} + e^{-ik\theta} = 2\cos k\theta$
$E = e^{i\theta}$	$\sqrt{z^2-1} = t$		$e^{ik\theta} - e^{-ik\theta} = 2i\operatorname{sen} k\theta$

Gabarito (incompleto e não revisado):

1) $$\begin{aligned}\int (2x+3)\sqrt{4x+5}\,dx &= \int (2(\tfrac{u}{4}-\tfrac{5}{4})+3)\sqrt{u}(\tfrac{1}{4})\,du \qquad \left[\begin{smallmatrix} u=4x+5 \\ x=\frac{u-5}{4} \\ dx=\frac{1}{4}du \end{smallmatrix}\right] \\ &= \int (\tfrac{1}{4})(\tfrac{2u}{4}-\tfrac{10}{4}-\tfrac{12}{4})\sqrt{u}\,du \\ &= \int (\tfrac{u}{8}-\tfrac{11}{8})\sqrt{u}\,du \\ &= \int \tfrac{1}{8}u^{3/2}-\tfrac{11}{8}u^{1/2}\,du \\ &= \tfrac{1}{8}\tfrac{5}{2}u^{5/2}-\tfrac{11}{8}\tfrac{2}{3}u^{3/2} \\ &= \tfrac{5}{16}u^{5/2}-\tfrac{11}{12}u^{3/2} \\ &= \tfrac{5}{16}(4x+5)^{5/2}-\tfrac{11}{12}(4x+5)^{3/2}\end{aligned}$$

2) $$\begin{aligned}&\int x^3\sqrt{1-x^2}\,dx \qquad \left[\begin{smallmatrix} s=x \end{smallmatrix}\right] \\ &= \int s^3\sqrt{1-s^2}\,ds \qquad \left[\begin{smallmatrix} s=\operatorname{sen}\theta \\ ds=\cos\theta d\theta \\ \sqrt{1-s^2}=\cos\theta \end{smallmatrix}\right] \\ &= \int (\operatorname{sen}\theta)^3\cos\theta\,\cos\theta\,ds \\ &= \int (\operatorname{sen}\theta)^2(\cos\theta)^2\operatorname{sen}\theta\,ds \qquad \left[\begin{smallmatrix} \cos\theta=c \\ (\operatorname{sen}\theta)^2=1-c^2 \\ \operatorname{sen}\theta d\theta=(-1)dc \end{smallmatrix}\right] \\ &= \int (1-c^2)c^2(-1)\,dc \\ &= \int c^4-c^2\,dc \\ &= \tfrac{1}{5}c^5-\tfrac{1}{3}c^3 \\ &= \tfrac{1}{5}\sqrt{1-s^2}^5-\tfrac{1}{3}\sqrt{1-s^2}^3 \\ &= \tfrac{1}{5}\sqrt{1-x^2}^5-\tfrac{1}{3}\sqrt{1-x^2}^3\end{aligned}$$

3) $$\begin{aligned}&(\operatorname{sen}5\theta)^2(\cos 6\theta)^2 \\ &= \left(\tfrac{E^5-E^{-5}}{2i}\right)^2\left(\tfrac{E^6+E^{-6}}{2}\right)^2 \\ &= -\tfrac{1}{16}(E^{10}-2+E^{-10})(E^{12}+2+E^{-12}) \\ &= -\tfrac{1}{16}\begin{pmatrix}(E^{10}-2+E^{-10})E^{12}+\\ (E^{10}-2+E^{-10})\cdot 2+\\ (E^{10}-2+E^{-10})E^{-12}\end{pmatrix} \\ &= -\tfrac{1}{16}\begin{pmatrix}E^{22}-2E^{12}+E^2+\\ 2E^{10}-4+2E^{-10}+\\ E^{-2}-2E^{-12}+E^{-22}\end{pmatrix} \\ &= -\tfrac{1}{16}((E^{22}+E^{-22})-2(E^{12}+E^{-12})+2(E^{10}+E^{-10})+(E^2+E^{-2})-4) \\ &= -\tfrac{1}{16}(2\cos 22\theta-4\cos 12\theta+4\cos 10\theta+2\cos 2\theta-4) \\ &= -\tfrac{1}{8}\cos 22\theta+\tfrac{1}{4}\cos 12\theta-\tfrac{1}{4}\cos 10\theta-\tfrac{1}{8}\cos 2\theta\tfrac{1}{4}\end{aligned}$$

$$\begin{aligned}\int (\operatorname{sen}5x)^2(\cos 6x)^2\,dx &= \int -\tfrac{1}{8}\cos 22x+\tfrac{1}{4}\cos 12x-\tfrac{1}{4}\cos 10x-\tfrac{1}{8}\cos 2x+\tfrac{1}{4}\,dx \\ &= -\tfrac{1}{8\cdot 22}\operatorname{sen}22x+\tfrac{1}{4\cdot 12}\operatorname{sen}12x-\tfrac{1}{4\cdot 10}\operatorname{sen}10x-\tfrac{1}{8\cdot 2}\operatorname{sen}2x+\tfrac{1}{4}x\end{aligned}$$

$$
\begin{aligned}
4)\quad \int \frac{x^3}{x^2+7x+12}\,dx &= \int x-7+\frac{37x+84}{x^2+7x+12}\,dx \\
&= \int x-7+\frac{37x+84}{(x+3)(x+4)}\,dx \\
&= \int x-7-\frac{27}{x+3}+\frac{64}{x+4}\,dx \\
&= \frac{x^2}{2}-7x+27\ln|x+3|+64\ln|x+4|
\end{aligned}
$$