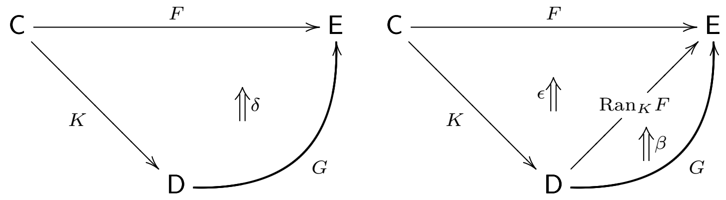
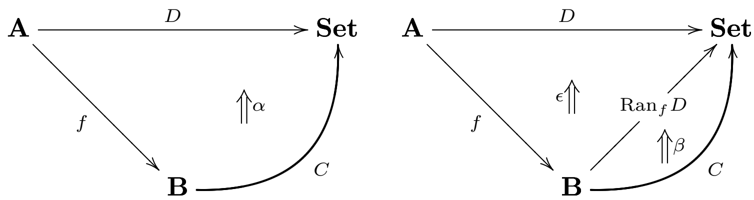
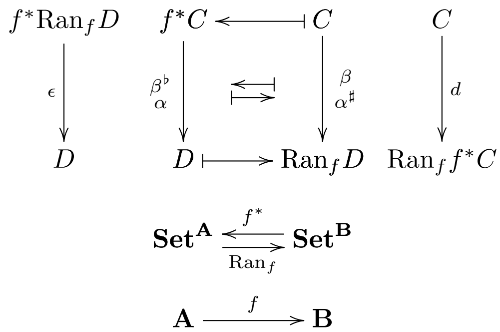
In [Rie16], sec.6.1, right Kan extensions are explained using the two diagrams below. The notation of cells is explained in sec.1.7 of the book, and modulo the types — that can be inferred from the diagrams — a right Kan extension of K along K is a pair $(\mathrm{Ran}_K F, \epsilon)$ such that for all (G, α) there is a unique β making everything commute.



If we specialize E to **Set** and do some renamings, the diagram becomes:



and if we change its *shape* to stress that ϵ "looks like" a counit map and Ran_f "looks like" the right adjoint to the functor f^*, we get this:



When the categories $\mathbf{A}$ and $\mathbf{B}$ are finite posets we get: 1) $\mathbf{Set}^{\mathbf{A}}$ and $\mathbf{Set}^{\mathbf{B}}$ are toposes; 2) the functor "precomposition with f", f^*, is very easy to define and to visualize, 3) the left and right Kan extensions Lan_f and Ran_f and can be defined and calculated by the formulas in sec.6.2 of [Rie16], 4) we have adjunctions $\mathrm{Lan}_f \dashv f^* \dashv \mathrm{Ran}_f$, and the structure $(\mathrm{Lan}_f \dashv f^* \dashv \mathrm{Ran}_f)$ can be seen as an essential geometric morphism $f : \mathbf{Set}^{\mathbf{A}} \to \mathbf{Set}^{\mathbf{B}}$ ([**Elephant1**], A4.1.4)

In [Rie16], sec.6.1, right Kan extensions are defined as this.

Given functors $F : \mathsf{C} \to \mathsf{E}$, $K : \mathsf{C} \to \mathsf{D}$, a right Kan extension of F along K is a functor $\mathrm{Ran}_K F : \mathsf{D} \to \mathsf{E}$ together with a natural transformation $\epsilon : (K; \mathrm{Ran}_K F) \Rightarrow F$ such that every pair $(G : D \to E, \delta : F \Rightarrow (K; G))$ factors uniquely through ϵ in this sense: there exists a unique $\alpha : G \Rightarrow \mathrm{Ran}_K F$

as illustrated.

For every $\alpha : f^* F \to G$

there is a unique $\beta : F \to f_* G$

such that $(f^* \beta; \epsilon) = \alpha$: