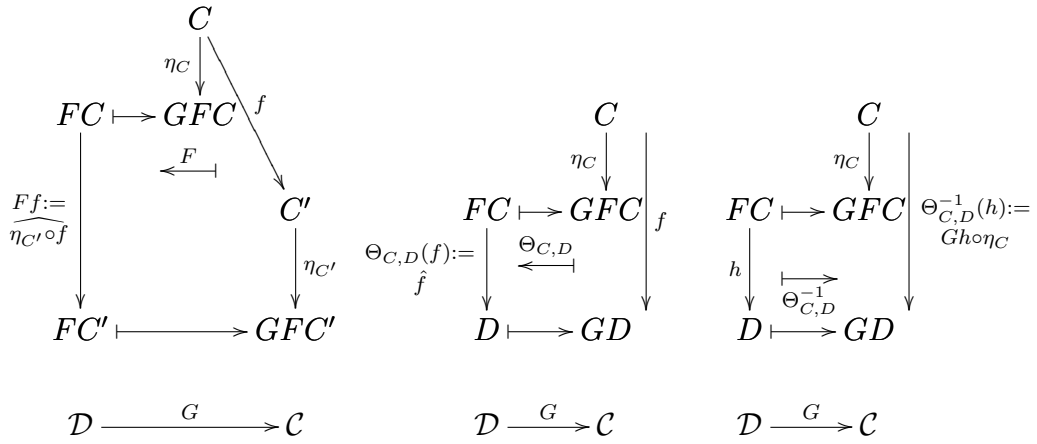
Notes on Samson Abramsky and Nikos Tzevelekos's
"Introduction to Categories and Categorical Logic" (2011):
https://arxiv.org/abs/1102.1313
https://arxiv.org/pdf/1102.1313.pdf

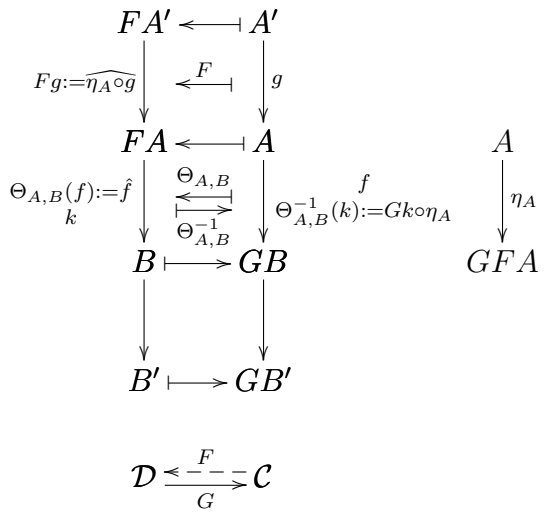
These notes are at:
http://angg.twu.net/LATEX/2020abramsky-tzevelekos.pdf

(P.48):

Proposition 69: universals define adjunctions.

Let $G : \mathcal{D} \to \mathcal{C}$ and suppose that we have an operation $C \mapsto (FC, \eta_C)$ that returns universal arrows. Then we can define an adjunction (F, G, Θ) from that:





$$\begin{pmatrix} F, \\ G, \\ \Theta, \\ \Theta^{-1} \end{pmatrix} := \begin{pmatrix} \begin{pmatrix} C \mapsto FC \\ (f : C \to C') \mapsto \eta_{C'} \circ f \end{pmatrix} \\ G, \\ \lambda C.\lambda D.\lambda (f : C \to GD).\hat{f}, \\ \lambda C.\lambda D.\lambda (h : FC \to D).Gh \circ \eta_C \end{pmatrix}$$