

Notes on Samson Abramsky and Nikos Tzevelekos's  
"Introduction to Categories and Categorical Logic" (2011):

<https://arxiv.org/abs/1102.1313>

<https://arxiv.org/pdf/1102.1313.pdf>

These notes are at:

<http://angg.twu.net/LATEX/2020abramsky-tzevelekos.pdf>

(P.48):

Proposition 69: universals define adjunctions.

Let  $G : \mathcal{D} \rightarrow \mathcal{C}$  and suppose that we have an operation  $C \mapsto (FC, \eta_C)$  that returns universal arrows. Then we can define an adjunction  $(F, G, \Theta)$  from that:

$$\begin{array}{c}
 \begin{array}{ccc}
 & C & \\
 & \eta_C \downarrow & \searrow f \\
 FC & \mapsto & GFC \\
 \downarrow \widehat{Ff} := \widehat{\eta_{C'} \circ f} & \longleftarrow F & \downarrow \eta_{C'} \\
 FC' & \mapsto & GFC' \\
 \mathcal{D} & \xrightarrow{G} & \mathcal{C}
 \end{array}
 & \Theta_{C,D}(f) := \hat{f} & \begin{array}{ccc}
 & C & \\
 & \eta_C \downarrow & \downarrow f \\
 FC & \mapsto & GFC \\
 \downarrow h & \longleftarrow \Theta_{C,D} & \downarrow \\
 D & \mapsto & GD \\
 \mathcal{D} & \xrightarrow{G} & \mathcal{C}
 \end{array}
 & \Theta_{C,D}^{-1}(h) := Gh \circ \eta_C & \begin{array}{ccc}
 & C & \\
 & \eta_C \downarrow & \downarrow \\
 FC & \mapsto & GFC \\
 \downarrow h & \longleftarrow \Theta_{C,D}^{-1} & \downarrow \\
 D & \mapsto & GD \\
 \mathcal{D} & \xrightarrow{G} & \mathcal{C}
 \end{array}
 \end{array}$$
  

$$\begin{array}{ccc}
 \begin{array}{ccc}
 FA' & \longleftarrow & A' \\
 \downarrow Fg := \widehat{\eta_A \circ g} & \longleftarrow F & \downarrow g \\
 FA & \longleftarrow & A \\
 \downarrow \Theta_{A,B}(f) := \hat{f} & \longleftarrow \Theta_{A,B} & \downarrow f \\
 B & \mapsto & GB \\
 \downarrow & & \downarrow \\
 B' & \mapsto & GB' \\
 \mathcal{D} & \xrightleftharpoons[F]{F} & \mathcal{C}
 \end{array}
 & \Theta_{A,B}^{-1}(k) := Gk \circ \eta_A & \begin{array}{c}
 A \\
 \downarrow \eta_A \\
 GFA
 \end{array}
 & \left( \begin{array}{c}
 F, \\
 G, \\
 \Theta, \\
 \Theta^{-1}
 \end{array} \right) := \\
 & & \left( \begin{array}{c}
 C \mapsto FC \\
 ((f : C \rightarrow C') \mapsto \eta_{C'} \circ f) \\
 G, \\
 \lambda C. \lambda D. \lambda (f : C \rightarrow GD). \hat{f}, \\
 \lambda C. \lambda D. \lambda (h : FC \rightarrow D). Gh \circ \eta_C
 \end{array} \right)
 \end{array}$$