Notes on Steve Awodey's "Category Theory":
https://www.andrew.cmu.edu/user/awodey/

These notes are at:
http://angg.twu.net/LATEX/2020awodey.pdf
http://angg.twu.net/math-b.html#notes-on-notation-2020

8.2 The Yoneda Embedding

Page 161:

Definition 8.1. The Yoneda embedding is the functor $y : \mathbf{C} \to \mathbf{Sets}^{\mathbf{C}^{\mathrm{op}}}$ taking $C \in \mathbf{C}$ to the contravariant representable functor, $yC = \mathrm{Hom}_{\mathbf{C}}(-, C) : \mathbf{C}^{\mathrm{op}} \to \mathbf{Sets}$ and taking $f : C \to D$ to the natural transformation, $yf = \mathrm{Hom}_{\mathbf{C}}(-, f) : \mathrm{Hom}_{\mathbf{C}}(-, C) \to \mathrm{Hom}_{\mathbf{C}}(-, D)$.

$$\begin{array}{ccc} & & 1 \\ & & \downarrow {\ulcorner f \urcorner} \\ C & \longmapsto & \mathrm{Hom}_{\mathbf{C}}(C, D) \\ \mathbf{C}^{\mathrm{op}} & \xrightarrow{\mathrm{Hom}_{\mathbf{C}}(-,D)} & \mathbf{Sets} \end{array}$$

$$\begin{array}{ccccc} yC & = & \mathrm{Hom}_{\mathbf{C}}(-, C) & \longrightarrow & \mathrm{Hom}_{\mathbf{Sets}}(1, \mathrm{Hom}_{\mathbf{C}}(-, D)) \\ & & & \searrow^{yf} & \updownarrow \cong \\ & & & & \mathrm{Hom}_{\mathbf{C}}(-, D) \ = \ yD \end{array}$$

$$\mathbf{Sets}^{\mathbf{C}^{\mathrm{op}}}$$

8.3 The Yoneda Lemma

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Lemma 8.2 (Yoneda). Let $\mathbf{C}$ be locally small. For any object $C \in \mathbf{C}$ and functor $F \in \mathbf{Sets}^{\mathbf{C}^{\mathrm{op}}}$ there is an isomorphism $\mathrm{Hom}(yC, F) \cong FC$ which, moreover, is natural in both F and C.

$$
\begin{array}{ccc}
 & & 1 \\
 & & \downarrow{\scriptstyle \ulcorner a \urcorner} \\
C & \longmapsto & FC
\end{array}
$$

$$
\mathbf{C}^{\mathrm{op}} \xrightarrow{\;F\;} \mathbf{Sets}
$$

$$
\begin{array}{ccc}
yC \;=\; \mathrm{Hom}_{\mathbf{C}}(-, C) & \longrightarrow & \mathrm{Hom}_{\mathbf{Sets}}(1, F-) \\
 & \searrow^{\vartheta} & \updownarrow{\scriptstyle \cong} \\
 & & F
\end{array}
$$

$$
\mathbf{Sets}^{\mathbf{C}^{\mathrm{op}}}
$$

9. Adjoints

(Page 180):

Definition 9.1 (preliminary). An adjunction between categories **C** and **D** consists of functors

$$F : \mathbf{C} \leftrightarrows \mathbf{D} : U$$

and a natural transformation $\eta : 1_C \to U \circ F$ with the UMP $\forall C.\forall D.\forall f.\exists! g.\_$ below.

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9.2 Hom-set definition

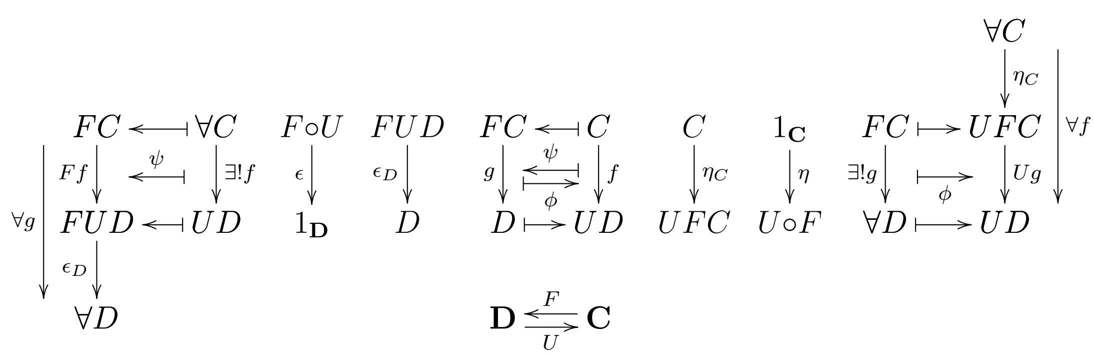
Proposition 9.4. Given categories and functors **C**, **D**, F, U, the following conditions are equivalent:

1. F is left adjoint to U; that is, there is a natural transformation $\eta : 1_C \to U \circ F$ with the UMP $\forall C.\forall D.\forall f.\exists! g.\_$ below;

2. For any $C \in \mathbf{C}$ and $D \in \mathbf{D}$ there is an isomorphism

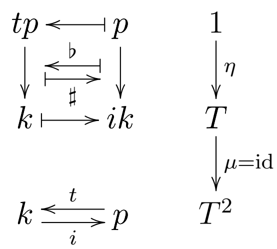
$$\varphi : \mathrm{Hom}_{\mathbf{D}}(FC, D) \to \mathrm{Hom}_{\mathbf{C}}(C, UD)$$

   that is natural in both C and D.



10. Monads and algebras

Example 10.4 (p.228): Let P be a poset.



Example 10.5: $(\mathcal{P}, \{-\}, \bigcup)$ on **Sets**

Proposition 10.6: Eilenberg-Moore