Notes on Bell's "Toposes and Local Set Theories" (Oxford, 1988).

These notes are at:
http://angg.twu.net/LATEX/2020bell-lst.pdf

See:
http://angg.twu.net/LATEX/2020favorite-conventions.pdf
http://angg.twu.net/math-b.html#favorite-conventions
I wrote these notes mostly to test if the conventions above are good enough.

3. Local Set Theories

(Page 70):

(L1)	$\alpha \Leftrightarrow \beta$	for	$\alpha = \beta$
(L2)	$true$	for	$* = *$
(L3)	$\alpha \wedge \beta$	for	$\langle \alpha, \beta \rangle = \langle true, true \rangle$
(L4)	$\alpha \Rightarrow \beta$	for	$(\alpha \wedge \beta) \Leftrightarrow \alpha$
(L5)	$\forall x \alpha$	for	$\{ x : \alpha \} = \{ x : true \}$
(L6)	$false$	for	$\forall \omega . \omega$
(L7)	$\neg \alpha$	for	$\alpha \Rightarrow false$
(L8)	$\alpha \vee \beta$	for	$\forall \omega [(\alpha \Rightarrow \omega \wedge \beta \Rightarrow \omega) \Rightarrow \omega]$
(L9)	$\exists x \alpha$	for	$\forall \omega [\forall x (\alpha \Rightarrow \omega) \Rightarrow \omega]$

(Page 71):

Basic axioms (left) and rules of inference (right):

$$\frac{}{\alpha : \alpha}\ \text{Taut} \qquad \frac{\Gamma : \alpha}{\beta, \Gamma : \alpha}\ \text{Thin}$$

$$\frac{}{: x_1 = *}\ \text{Unit} \qquad \frac{\Gamma : \alpha \quad \alpha, \Gamma : \beta}{\Gamma : \beta}\ \text{Cut}$$

$$\frac{}{x = y, \alpha(z/x) : \alpha(z/y)}\ \text{Equa} \qquad \frac{\Gamma : \alpha}{\Gamma(x/\tau) : \alpha(x/\tau)}\ \text{Subst}$$

$$\frac{}{: (\langle x_1, \ldots, x_n \rangle)_i = x_i}\ \text{Produ} \qquad \frac{\Gamma : x \in \sigma \Leftrightarrow x \in \tau}{\Gamma : \sigma = \tau}\ \text{Ext}$$

$$\frac{}{: x = \langle (x)_1, \ldots, (x)_n \rangle}\ \text{Produ} \qquad \frac{\alpha, \Gamma : \beta \quad \beta, \Gamma : \alpha}{\Gamma : \alpha \Leftrightarrow \beta}\ \text{Equiv}$$

$$\frac{}{: x \in \{\, x : \alpha \,\} \Leftrightarrow \alpha}\ \text{Compr}$$

(Page 71):
(3.1.5) (i) and (ii):

$$\frac{true, \alpha : \alpha \quad \dfrac{}{\alpha, \alpha : true}}{\alpha : \alpha = true} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\alpha : \alpha}\ \text{Taut}}{true, \alpha : \alpha}\ \text{Thin} \quad \dfrac{\dfrac{\dfrac{}{: * = *}\ \text{Unit}}{: true}\ \text{(L2)}}{\alpha : true}\ \text{Thin}}{\alpha : \alpha \Leftrightarrow true}\ \text{Equiv}}{\alpha : \alpha = true}\ \text{(L1)}$$

$$\frac{\dfrac{\omega = \omega', \omega' : \omega}{\alpha = true, true : \alpha} \quad \dfrac{}{: true}}{\alpha = true : \alpha} \quad \rightsquigarrow \quad \frac{\dfrac{}{: true} \quad \dfrac{\dfrac{}{\omega = \omega', \omega' : \omega}\ \text{Equa}}{\alpha = true, true : \alpha}\ \text{Subst}}{true, \alpha = true : \alpha}}{\alpha = true : \alpha}\ \text{Cut}$$