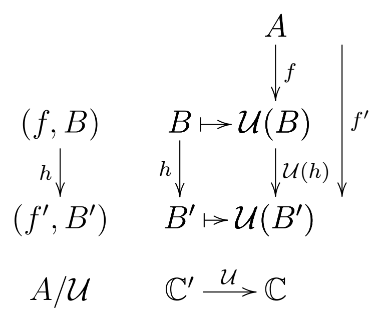
Notes on Razvan Diaconescu's
"Institution-independent Model Theory" (Birkhäuser, 2008)
http://www.springer.com/birkhauser/mathematics/book/978-3-7643-8707-5

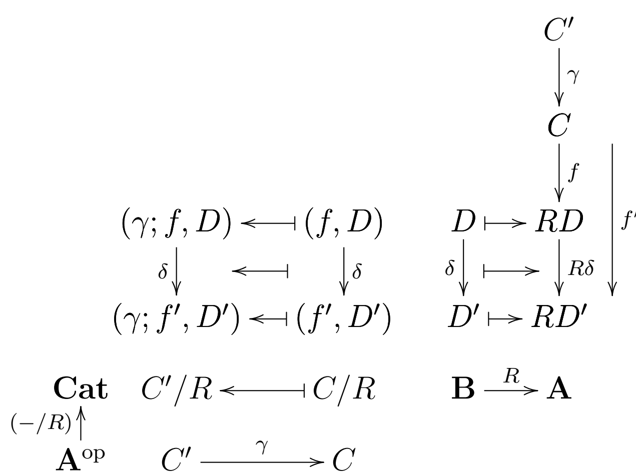
These notes are at:
http://angg.twu.net/LATEX/2020institutions.pdf

Comma categories

(Page 11):

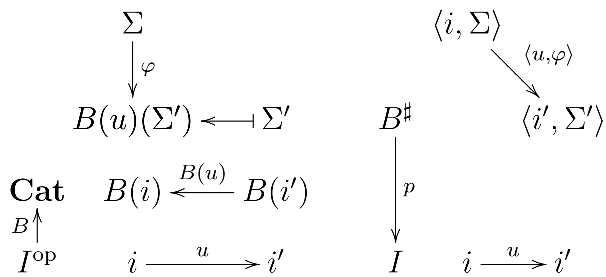


The construction of the functor $C \mapsto C/R$, that we will use in page 45:



2.5. Indexed Categories and Fibrations

(Page 20):
The Grothendieck category $B^\sharp$ of $B : I^{\mathrm{op}} \to \mathbf{Cat}$:

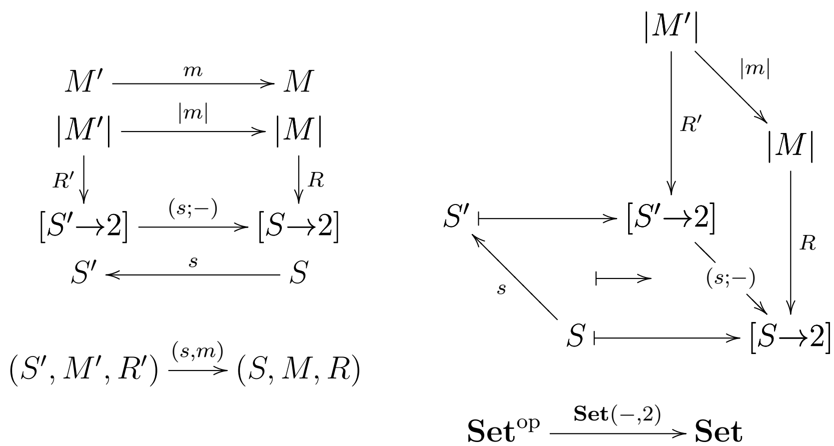


3.4. Institutions as functors

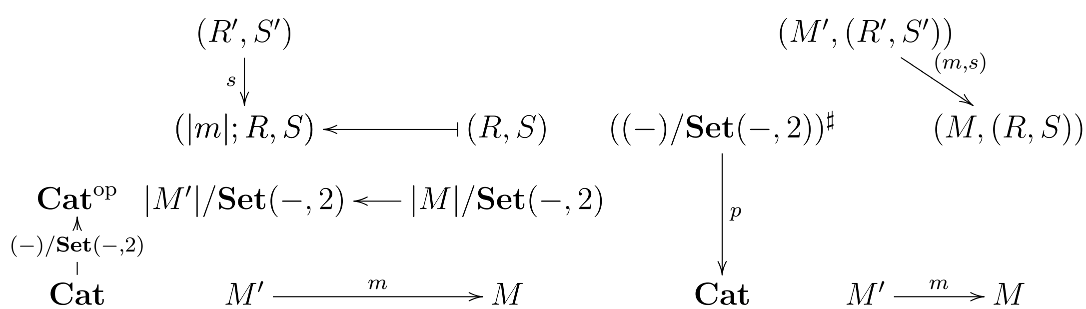
(Page 45):

A room is a triple (S, M, R) where: $\left(\begin{array}{c} S \text{ is a set,} \\ M \text{ is a category,} \\ R : |M| \to \mathbf{Set}(S, 2) \end{array}\right)$.

A room morphism $(s, m) : (S', M', R') \to (S, M, R)$ is:



A room morphism is a morphism in the Grothendieck category blah:



$$
\begin{array}{ccc}
(|m|;R,\ S) & \longleftarrow\!\shortmid & (R,S) \\
{\scriptstyle s}\uparrow & & \uparrow{\scriptstyle s} \\
(|m|;R',\ S') & \longleftarrow\!\shortmid & (R',S')
\end{array}
\qquad
\begin{array}{cccc}
 & & |M'| & \\
 & & \downarrow{\scriptstyle |m|} & \\
 & & |M| & \\
 & & \downarrow{\scriptstyle R} & \\
S & \longmapsto & \mathbf{Set}(S,2) & \Big\downarrow{\scriptstyle R'} \\
{\scriptstyle s}\uparrow & & \downarrow{\scriptstyle (s;-)} & \\
S' & \longmapsto & \mathbf{Set}(S',2) &
\end{array}
$$

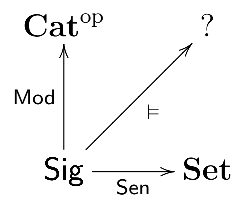
$$
\begin{array}{ccc}
\mathbf{Cat} & & |M'|/\mathbf{Set}(-,2) \longleftarrow\!\shortmid |M|/\mathbf{Set}(-,2) \\
\uparrow{\scriptstyle (|-|)/\mathbf{Set}(-,2)} & & |M'| \xrightarrow{|m|} |M| \\
\mathbf{Cat}^{\mathrm{op}} & & M' \xrightarrow{m} M
\end{array}
\qquad
\mathbf{Set}^{\mathrm{op}} \xrightarrow{\mathbf{Set}(-,2)} \mathbf{Set}
$$

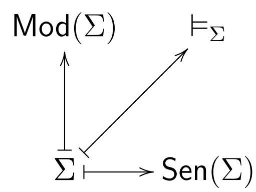
Typing institutions

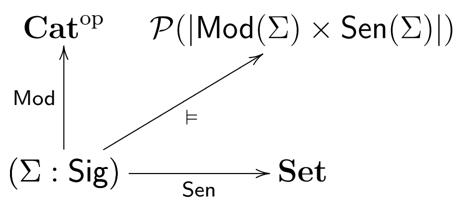
$$\begin{array}{rcl} \mathsf{Sig} & \text{is} & \text{a category} \\ \mathsf{Sen} & : & \mathsf{Sig} \to \mathbf{Set} \\ \mathsf{Mod} & : & \mathsf{Sig} \to \mathbf{Cat}^{\mathrm{op}} \\ \models & : & ? \\ \models & : & (\Sigma : |\mathsf{Sig}|) \to \mathcal{P}(|\mathsf{Mod}(\Sigma)| \times \mathsf{Sen}(\Sigma)) \end{array}$$

$$\begin{array}{rcl} \Sigma & \in & |\mathsf{Sig}| \\ \models_\Sigma & \subseteq & |\mathsf{Mod}(\Sigma)| \times \mathsf{Sen}(\Sigma) \\ \models_\Sigma & \in & \mathcal{P}(|\mathsf{Mod}(\Sigma)| \times \mathsf{Sen}(\Sigma)) \\ \models & \in & (\Sigma : |\mathsf{Sig}|) \to \mathcal{P}(|\mathsf{Mod}(\Sigma)| \times \mathsf{Sen}(\Sigma)) \\ \models & \in & \Pi\Sigma{:}|\mathsf{Sig}|.\mathcal{P}(|\mathsf{Mod}(\Sigma)| \times \mathsf{Sen}(\Sigma)) \end{array}$$

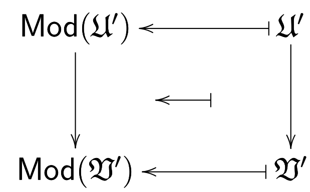
Drawing institutions (1)

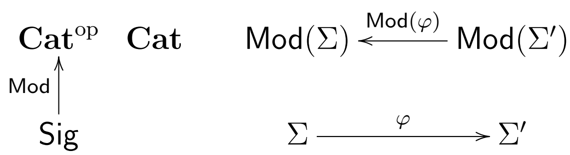






Drawing institutions (2)
Gaina-Kowalski, p.3:





Drawing institutions (3)

$$e \longmapsto \mathsf{Sen}(\varphi)(e)$$

$$\begin{array}{ccc} \mathbf{Set} & \quad & \mathsf{Sen}(\Sigma) \xrightarrow{\mathsf{Sen}(\varphi)} \mathsf{Sen}(\Sigma') \\ \uparrow{\scriptstyle \mathsf{Sen}} & & \\ \mathsf{Sig} & & \Sigma \xrightarrow{\quad\varphi\quad} \Sigma' \end{array}$$

Drawing institutions (4)