Notes on Emily Riehl's "Categories in Context" (2016):
http://www.math.jhu.edu/~eriehl/
http://www.math.jhu.edu/~eriehl/context/
http://www.math.jhu.edu/~eriehl/context.pdf

These notes are at:
http://angg.twu.net/LATEX/2020riehl.pdf

See:
http://angg.twu.net/LATEX/2020favorite-conventions.pdf
http://angg.twu.net/math-b.html#favorite-conventions
I wrote these notes mostly to test if the conventions above are good enough.

2.2. The Yoneda Lemma

2.2.9. Corollary: matrices

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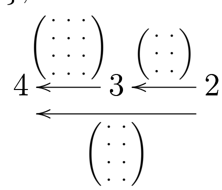
Corollary 2.2.9. Every row operation on matrices with n rows is defined by left multiplication by some $n \times n$ matrix, namely the matrix obtained by performing the row operation on the identity matrix.

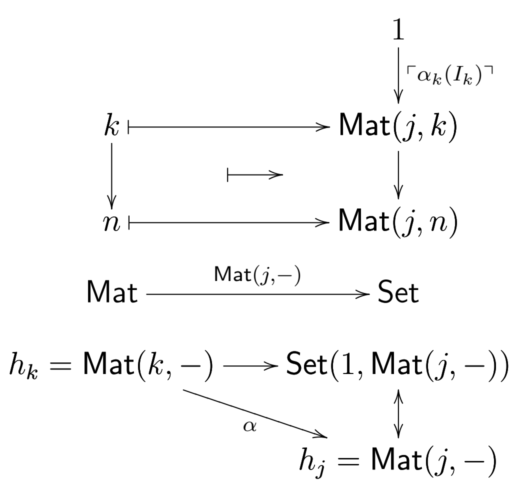
She gave a presentation about this in the Tutorial Day of the ACT2020:
http://www.math.jhu.edu/~eriehl/matrices.pdf
https://www.youtube.com/watch?v=SsgEvrDFJsM

Let Mat be the category that has:
$\mathsf{Objs}(\mathsf{Mat}) = \{1, 2, 3, \ldots\}$,
$\mathrm{Hom}_{\mathsf{Mat}}(k, m) = \mathsf{Mat}(k, m) = \{\, m \times k \text{ -matrices} \,\}$,

and composition like this: $n \xleftarrow{A} m \xleftarrow{B} k$, with $n \xleftarrow{A \circ B = AB} k$



The main diagram is:



2.3. Universal properties and universal elements

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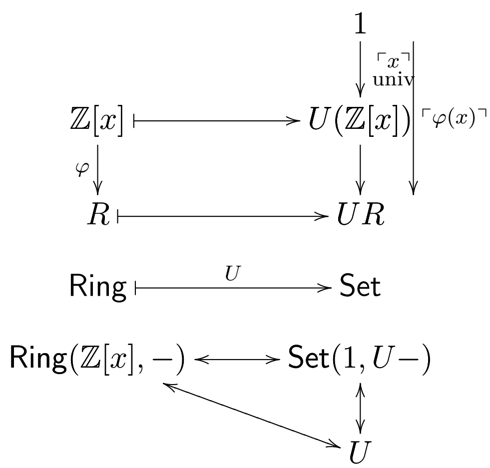
Example 2.3.4. Recall from Example 2.1.5(iv) that the forgetful functor $U : \mathsf{Ring} \to \mathsf{Set}$ is represented by the ring $\mathbb{Z}[x]$. The universal element, which defines the natural isomorphism

$$\mathsf{Ring}(\mathbb{Z}[x], R) \cong UR,$$

is the element $x \in \mathbb{Z}[x]$. As in the proof of the Yoneda lemma, the bijection (2.3.5) is implemented by evaluating a ring homomorphism $\varphi : \mathbb{Z}[x] \to R$ at the element $x \in Z[x]$ to obtain an element $\varphi(x) \in R$.



Example 2.3.7.

$$\begin{array}{ccc} & & 1 \\ & & \downarrow{\scriptstyle \ulcorner\otimes\urcorner \atop \text{univ}} \\ V\otimes_k W & \longmapsto & \mathsf{Bilin}(V,W;V\otimes_k W) \\ {\scriptstyle \bar f}\downarrow & & \downarrow \\ U & \longmapsto & \mathsf{Bilin}(V,W;U) \end{array} \qquad 1 \xrightarrow{\ulcorner f\urcorner} \mathsf{Bilin}(V,W;U)$$

$$\mathsf{Vect}_k \overset{U}{\longmapsto} \mathsf{Set}$$

$$\begin{array}{ccc} \mathsf{Vect}(V\otimes_k W,-) & \longleftrightarrow & \mathsf{Set}(1,\mathsf{Bilin}(V,W;-)) \\ & \searrow\!\!\!\nwarrow & \updownarrow \\ & & \mathsf{Bilin}(V,W;-) \end{array}$$

2.4 The category of elements

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2.4.1: Covariant:

$$\begin{array}{ccc} & & 1 \\ & & \downarrow{\scriptstyle \ulcorner x\urcorner} \\ c & \longmapsto & Fc \\ {\scriptstyle f}\downarrow & \longmapsto & \downarrow{\scriptstyle Ff} \\ c' & \longmapsto & Fc' \end{array} \quad 1 \xrightarrow{\ulcorner x'\urcorner} Fc' \qquad \begin{array}{ccc} (c,x) & \longmapsto & c \\ {\scriptstyle f}\downarrow & \longmapsto & \downarrow{\scriptstyle f} \\ (c',x') & \longmapsto & c' \end{array}$$

$$\mathsf{C} \xrightarrow{F} \mathsf{Set} \qquad \textstyle\int F \xrightarrow{\Pi} \mathsf{C}$$

2.4.2: Contravariant:

$$\begin{array}{ccc} & & 1 \\ & & \downarrow{\scriptstyle \ulcorner x'\urcorner} \\ c' & \longmapsto & Fc' \\ {\scriptstyle f}\uparrow & \longmapsto & \downarrow{\scriptstyle Ff} \\ c & \longmapsto & Fc \end{array} \quad 1 \xrightarrow{\ulcorner x\urcorner} Fc \qquad \begin{array}{ccc} (c',x') & \longmapsto & c' \\ {\scriptstyle f}\uparrow & \longmapsto & \uparrow{\scriptstyle f} \\ (c,x) & \longmapsto & c \end{array}$$

$$\mathsf{C}^{\text{op}} \xrightarrow{F} \mathsf{Set} \qquad \textstyle\int F \xrightarrow{\Pi} \mathsf{C}$$

4. Adjunctions

4.5.2. Right adjoints preserve limits

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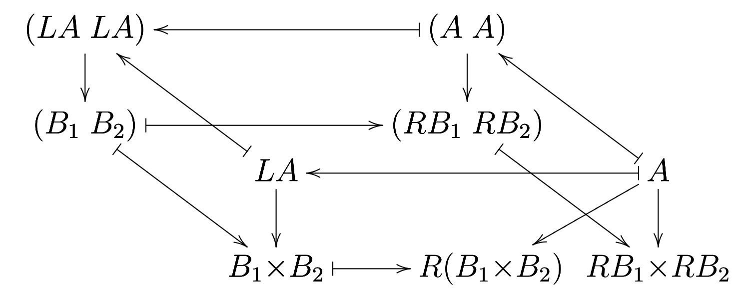

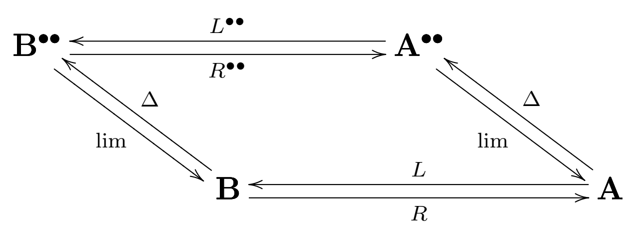

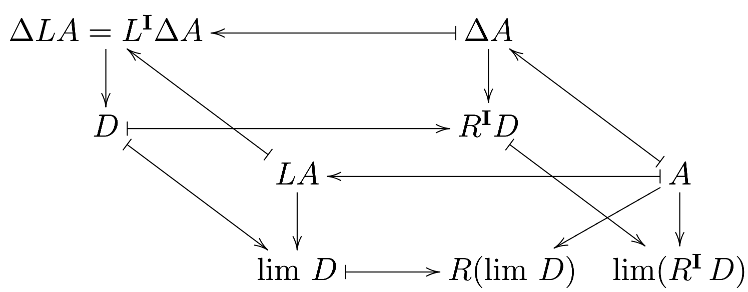

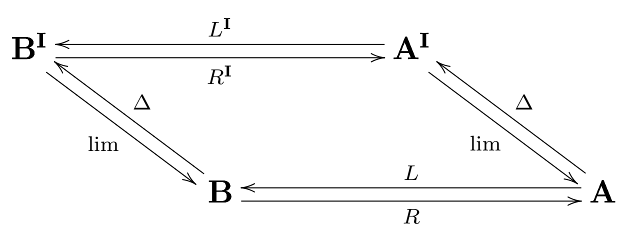

6.1. Kan Extensions

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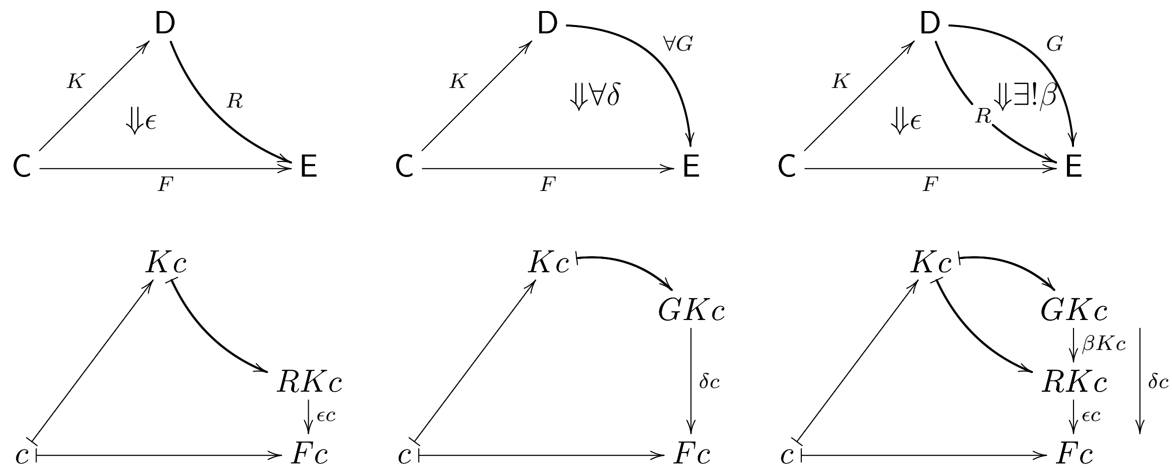
Definition 6.1.1 (second half). Given functors F and K a right kan extension of F along K is a pair (R, ϵ) such that $\forall G.\forall\delta.\exists!\beta.\ \delta = \epsilon \cdot \beta K$.

Lower half: internal view — $\delta = \epsilon \cdot \beta K$ becomes $\forall c.\ \delta c = \epsilon c \circ \beta K c$.



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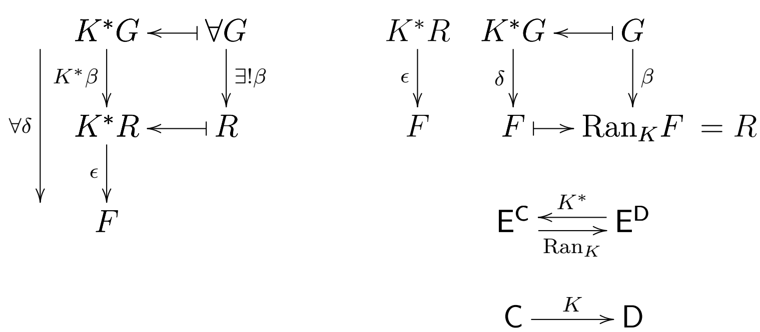
Proposition 6.1.5 (right Kan only). Fix K and suppose that we have an operation $F \mapsto (\mathrm{Ran}_K F, \epsilon)$ that returns right Kan extensions. We can use that to build an adjunction $K^* \dashv \mathrm{Ran}_K$, where K^* is pre-composition with K.



6.2 A formula for Kan extensions

Riehl's formula for $\mathrm{Ran}_K F(d)$ is:

$\mathrm{Ran}_K F(d) = \lim(d \downarrow K \xrightarrow{\Pi_d} \mathsf{C} \xrightarrow{F} \mathsf{E})$

I'll change the notation by $\begin{bmatrix} \mathsf{C}{:=}\mathbf{A} \\ \mathsf{D}{:=}\mathbf{B} \\ \mathsf{E}{:=}\mathbf{Set} \\ K{:=}F \\ F{:=}H \\ d{:=}B \end{bmatrix}$

$F_* H(B) = \mathrm{Ran}_F H(B) = \lim(B \downarrow F \xrightarrow{\pi_Q} \mathbf{A} \xrightarrow{F} \mathbf{Set})$

$$F : \mathbf{A} \to \mathbf{B} \qquad \text{is} \qquad \begin{pmatrix} & & 2 \\ & & \downarrow \\ 5 & \to & 6 \end{pmatrix} \xrightarrow{F} \begin{pmatrix} 1' & \to & 2' \\ \downarrow & & \downarrow \\ 3' & \to & 4' \\ \downarrow & & \downarrow \\ 5' & \to & 6' \end{pmatrix}$$

$$\begin{pmatrix} & & \left(\begin{smallmatrix} & 2 \\ 6 & F6 \end{smallmatrix}\right) \\ & & \downarrow \\ \left(\begin{smallmatrix} & 5 \\ 6 & F6 \end{smallmatrix}\right) & \longrightarrow & \left(\begin{smallmatrix} & 6 \\ 6 & F6 \end{smallmatrix}\right) \end{pmatrix}$$