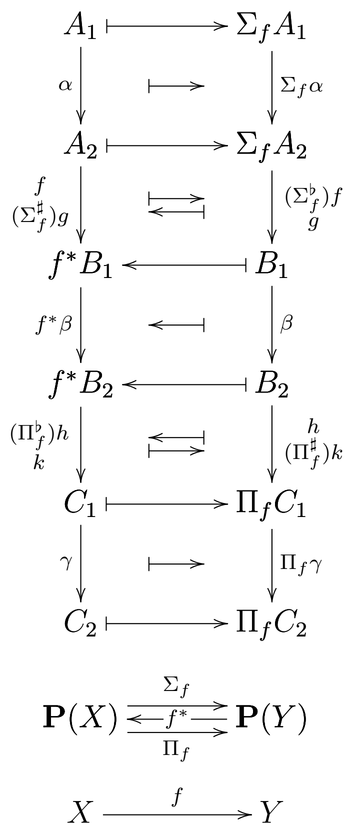
Adjoints to an arbitrary f^*:



Equality as an adjoint to Δ^*:

$$
\begin{array}{ccc}
A_1(x) & \longmapsto & x{=}x' \wedge A_1(x) \\
\alpha\downarrow & \longmapsto & \downarrow \Sigma_\Delta\alpha \\
A_2(x) & \longmapsto & x{=}x' \wedge A_2(x) \\
{\scriptstyle f}\;\;(\Sigma^\sharp_\Delta)g\downarrow & \rightleftarrows & \downarrow (\Sigma^\flat_\Delta)f\;\;{\scriptstyle g} \\
B_1(x,x) & \longleftarrow\!\!\!\mid & B_1(x,x') \\
\Delta^*\beta\downarrow & \longleftarrow\!\!\!\mid & \downarrow\beta \\
B_2(x,x) & \longleftarrow\!\!\!\mid & B_2(x,x') \\
(\Pi^\flat_\Delta)h\;\;{\scriptstyle k}\downarrow & \leftrightarrows & \downarrow {\scriptstyle h}\;\;(\Pi^\sharp_\Delta)k \\
C_1(x) & \longmapsto & x{=}x' \supset C_1(x) \\
\gamma\downarrow & \longmapsto & \downarrow\Pi_\Delta\gamma \\
C_2(x) & \longmapsto & x{=}x' \supset C_2(x)
\end{array}
$$

$$
\Sigma_\Delta\alpha := \left(\dfrac{x{=}x' \wedge A_1(x) \qquad \dfrac{\dfrac{x{=}x' \wedge A_1(x)}{A_1(x)}}{\begin{matrix}\vdots\,\alpha\\ \end{matrix}}}{\dfrac{x{=}x' \qquad A_2(x)}{x{=}x' \wedge A_2(x)}} \right)
$$

$$
(\Sigma^\sharp_\Delta)g := \left(\dfrac{\dfrac{\dfrac{}{x{=}x} \quad A_2(x)}{x{=}x \wedge A_2(x)} \qquad \begin{matrix}[x{=}x' \wedge A_2(x)]^1\\ \vdots\, g\\ B_1(x,x')\end{matrix}}{B_1(x,x)}\,[x' := x];1 \right)
$$

$$
(\Sigma^\flat_\Delta)f := \left(\dfrac{x{=}x' \wedge A_2(x) \qquad \begin{matrix}\dfrac{x{=}x' \wedge A_2(x)}{A_2(x)}\\ \vdots\, f\end{matrix}}{\dfrac{x{=}x' \qquad B_1(x,x)}{B_1(x,x')}} \right)
$$

$$
\Delta^*\beta := \left(\dfrac{B_1(x,x) \qquad \begin{matrix}[B_1(x,x')]^1\\ \vdots\,\beta\\ B_2(x,x')\end{matrix}}{B_2(x,x)}\,[x' := x];1 \right)
$$

$$
(\Pi^\flat_\Delta)h := \left(\dfrac{\dfrac{}{x{=}x} \qquad \dfrac{B_2(x,x) \quad \begin{matrix}[B_2(x,x')]^1\\ \vdots\, h\\ x{=}x' \supset C_1(x)\end{matrix}}{x{=}x \supset C_1(x)}\,[x' := x];1}{C_1(x)} \right)
$$

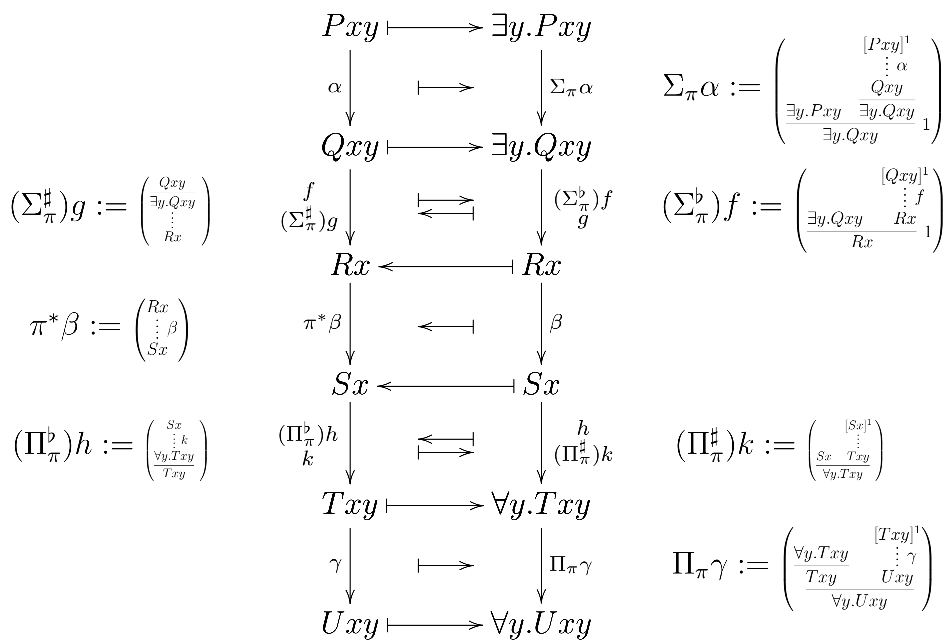
$$
(\Pi^\sharp_\Delta)k := \left(\dfrac{\dfrac{B_2(x,x') \qquad \dfrac{[x{=}x']^2 \qquad \dfrac{\begin{matrix}[B_2(x,x)]^1\\ \vdots\, k\\ C_1(x)\end{matrix}}{B_2(x,x) \supset C_1(x)}\,1}{B_2(x,x') \supset C_1(x)}}{C_1(x)}}{x{=}x' \supset C_1(x)} \right)
$$

$$
\Pi_\Delta\gamma := \left(\dfrac{\begin{matrix}\dfrac{[x{=}x']^1 \qquad x{=}x' \supset C_1(x)}{C_1(x)}\\ \vdots\,\gamma\\ C_2(x)\end{matrix}}{x{=}x' \supset C_2(x)}\,1 \right)
$$

$$
\mathbf{P}(X) \underset{\Pi_\Delta}{\overset{\Sigma_\Delta}{\underset{\xleftarrow{\Delta^*}}{\rightrightarrows}}} \mathbf{P}(X \times X)
$$

$$
\begin{array}{ccc}
x & \longmapsto & (x, x') \\
X & \xrightarrow{\Delta} & X \times X
\end{array}
$$

Quantifiers as adjoints to π^*:

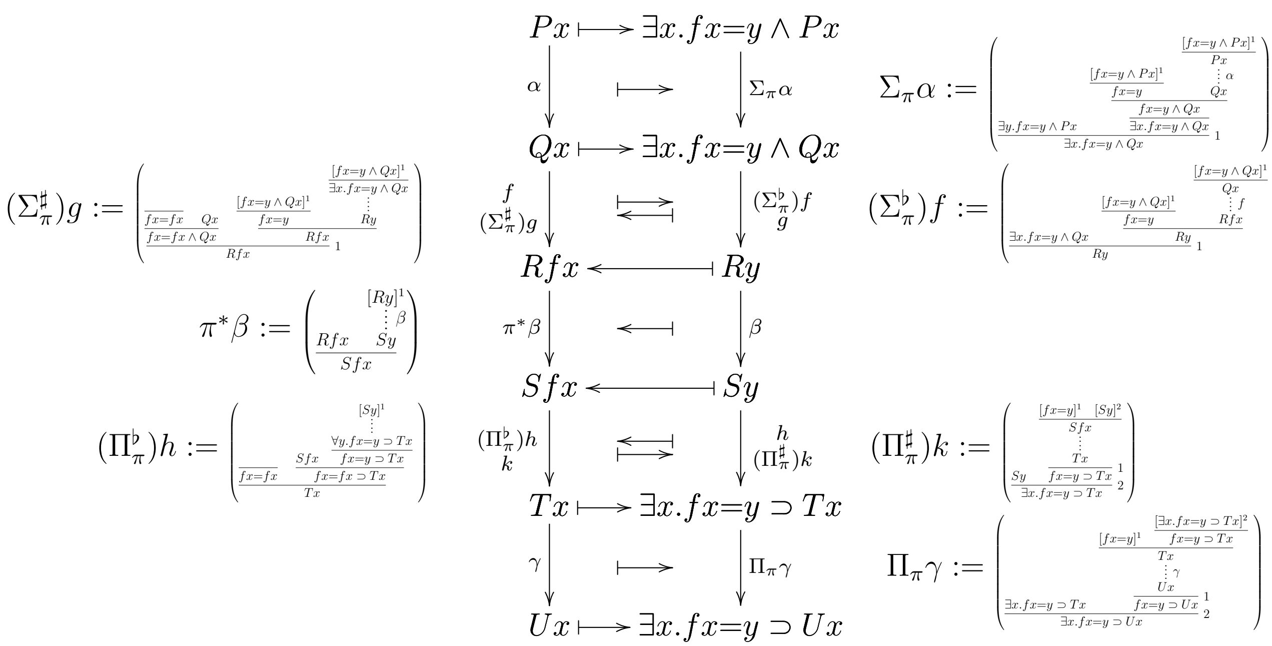


$$\mathbf{P}(X\times Y)\ \substack{\xrightarrow{\Sigma_\pi}\\ \xleftarrow{\pi^*}\\ \xrightarrow[\Pi_\pi]{}}\ \mathbf{P}(X)$$

$$(x,y) \longmapsto x$$

$$X\times Y \xrightarrow{\pi} X$$

Adjoints to $(y := f(x))^*$ can be built using quantifiers and equality:



$$\mathbf{P}(X) \underset{\Pi_f}{\overset{\Sigma_f}{\underset{\longleftarrow f^* \longrightarrow}{\longrightarrow}}} \mathbf{P}(Y)$$

$$x \longmapsto fx$$
$$X \xrightarrow{\pi} Y$$