

Tutorial world:

Level 1/4:

$$\frac{x, y, z: \text{mynat}}{\text{refl}: \llbracket x * y + z = x * y + z \rrbracket}$$

Level 2/4:

$$\frac{\text{refl}: \llbracket 2 * (x + 7) = 2 * (x + 7) \rrbracket \quad h: \llbracket y = x + 7 \rrbracket}{(\text{rw } h): \llbracket 2 * y = 2 * (x + 7) \rrbracket}$$

$$\frac{\text{refl}: \llbracket 2 * y = 2 * y \rrbracket \quad h: \llbracket y = x + 7 \rrbracket}{(\text{rw } \leftarrow h): \llbracket 2 * y = 2 * (x + 7) \rrbracket}$$

Level 3/4:

$$\frac{\text{refl}: \llbracket \text{succ } b = \text{succ } b \rrbracket \quad h: \llbracket \text{succ } a = b \rrbracket}{(\text{rw } h): \llbracket \text{succ}(\text{succ } a) = \text{succ } b \rrbracket}$$

$$\frac{\text{refl}: \llbracket \text{succ}(\text{succ } a) = \text{succ}(\text{succ } a) \rrbracket \quad h: \llbracket \text{succ } a = b \rrbracket}{(\text{rw } \leftarrow h): \llbracket \text{succ}(\text{succ } a) = \text{succ } b \rrbracket}$$

Level 4/4:

$$\frac{\text{refl}: \llbracket \text{succ } a = \text{succ } a \rrbracket \quad \text{add_zero}: \llbracket a + 0 = a \rrbracket \quad \text{add_succ}: \llbracket a + \text{succ } d = \text{succ}(a + d) \rrbracket}{(\text{rw } \text{add_zero}): \llbracket \text{succ}(a + 0) = \text{succ } a \rrbracket \quad \text{add_succ}: \llbracket a + \text{succ } 0 = \text{succ}(a + 0) \rrbracket}$$

$$(\text{rw } \text{add_succ}): \llbracket a + \text{succ } 0 = \text{succ } a \rrbracket$$

Note: this is a *very improvised* translation of the first exercises of the Natural Numbers Game to Natural Deduction. I started learning Lean in the beginning of oct/2021 and started to typeset these tress in 2021oct13, so this is very new and full of guesses — mainly of the kind “with the right abbreviations I can write this term in this way”.

Some links:

https://www.ma.imperial.ac.uk/~buzzard/xena/natural_number_game/
<http://math.andrej.com/2021/06/24/the-dawn-of-formalized-mathematics/>
<http://math.andrej.com/asset/data/the-dawn-of-formalized-mathematics.pdf>
<http://www.youtube.com/watch?v=fty9QL4aSRc> Vladislav Zavalov - Haskell to Core
<http://angg.twu.net/LATEX/2020favorite-conventions.pdf#page=35> [·]
<http://angg.twu.net/LATEX/2021lean-nng.pdf> this PDF
<http://angg.twu.net/math-b.html> my home page
<http://angg.twu.net/dednat6.html>

Addition world:

Level 1/6:

$$\frac{\frac{\text{refl}:\llbracket 0 = 0 \rrbracket}{(\text{rw add_zero}):\llbracket 0 + 0 = 0 \rrbracket} \quad \frac{\text{add_zero}:\llbracket a + 0 = a \rrbracket}{\text{add_zero}:\llbracket 0 + 0 = 0 \rrbracket} \quad \frac{\frac{\text{refl}:\llbracket \text{succ}(d) = \text{succ } d \rrbracket} \quad (\text{hd}:\llbracket 0 + d = d \rrbracket)^1}{(\text{rw hd}):\llbracket \text{succ}(0 + d) = \text{succ } d \rrbracket} \quad \frac{\text{add_succ}:\llbracket a + \text{succ } d = \text{succ}(a + d) \rrbracket}{\text{add_succ}:\llbracket 0 + \text{succ } d = \text{succ}(0 + d) \rrbracket}}{(\text{induction n with d hd}):\llbracket 0 + n = n \rrbracket} \quad 1$$