Cálculo 3 - 2022.1

P2 (Segunda prova)

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Questão 1
(Total: 1.0 pts)
Digamos que:

$$\begin{aligned} F(x,y) &= a \\ &+ bx + cy \\ &+ dx^2 + exy + fy^2 \end{aligned}$$

a) **(0.2 pts)** Calcule F_x, F_y, F_{xx}, F_{xy} e F_{yy} nos pontos (x,y) e $(0,0)$.

b) **(0.8 pts)** Mostre como reescrever $F(x,y)$ como

$$\begin{aligned} F(x,y) &= \_\_\_ \\ &+ \_\_\_x + \_\_\_y \\ &+ \_\_\_x^2 + \_\_\_xy + \_\_\_y^2 \end{aligned}$$

onde em cada lacuna você vai pôr uma expressão que depende só das derivadas parciais de $F(x,y)$ no ponto $(0,0)$.

Questão 2
(Total: 3.0 pts)
Digamos que:

$$\begin{aligned} G(x_0+\Delta x, y_0+\Delta y) &= a \\ &+ b\Delta x + c\Delta y \\ &+ d(\Delta x)^2 + e\Delta x\Delta y + f(\Delta y)^2 \end{aligned}$$

a) **(0.6 pts)** Calcule G_x, G_y, G_{xx}, G_{xy} e G_{yy} nos pontos (x,y) e $(0,0)$.

b) **(2.4 pts)** Mostre como reescrever $G(x,y)$ como

$$\begin{aligned} G(x,y) &= \_\_\_ \\ &+ \_\_\_\Delta x + \_\_\_\Delta y \\ &+ \_\_\_\Delta x^2 + \_\_\_\Delta x\Delta y + \_\_\_\Delta y^2 \end{aligned}$$

onde em cada lacuna você vai pôr uma expressão que depende só das derivadas parciais de $G(x,y)$ no ponto (x_0,y_0).

Gabarito das questões 1 e 2

$$\begin{aligned}
F(x,y) &= a + bx + cy + dx^2 + exy + fy^2 \\
F_x(x,y) &= b + 2dx + ey \\
F_{xx}(x,y) &= 2d \\
F_{xy}(x,y) &= e \\
F_y(x,y) &= c + ex + 2fy \\
F_{yx}(x,y) &= e \\
F_{yy}(x,y) &= 2f
\end{aligned}$$

$$\begin{aligned}
F(x_0+\Delta x, y_0+\Delta y) &= a + b\Delta x + c\Delta y + d\Delta x^2 + e\Delta x\Delta y + f\Delta y^2 \\
F_x(x_0+\Delta x, y_0+\Delta y) &= b + 2d\Delta x + e\Delta y \\
F_{xx}(x_0+\Delta x, y_0+\Delta y) &= 2d \\
F_{xy}(x_0+\Delta x, y_0+\Delta y) &= e \\
F_y(x_0+\Delta x, y_0+\Delta y) &= c + e\Delta x + 2f\Delta y \\
F_{yx}(x_0+\Delta x, y_0+\Delta y) &= e \\
F_{yy}(x_0+\Delta x, y_0+\Delta y) &= 2f
\end{aligned}$$

$$\begin{aligned}
F(0,0) &= a \\
F_x(0,0) &= b \\
F_{xx}(0,0) &= 2d \\
F_{xy}(0,0) &= e \\
F_y(0,0) &= c \\
F_{yx}(0,0) &= e \\
F_{yy}(0,0) &= 2f
\end{aligned}$$

$$\begin{aligned}
F(x_0,y_0) &= a \\
F_x(x_0,y_0) &= b \\
F_{xx}(x_0,y_0) &= 2d \\
F_{xy}(x_0,y_0) &= e \\
F_y(x_0,y_0) &= c \\
F_{yx}(x_0,y_0) &= e \\
F_{yy}(x_0,y_0) &= 2f
\end{aligned}$$

$$\begin{aligned}
F(x,y) &= F(0,0) \\
&+ F_x(0,0)x + F_y(0,0)y \\
&+ \tfrac{1}{2}F_{xx}(0,0)x^2 + F_{xy}(0,0)xy + \tfrac{1}{2}F_{yy}(0,0)y^2
\end{aligned}$$

$$\begin{aligned}
F(x_0+\Delta x, y_0+\Delta y) &= F(x_0,y_0) \\
&+ F_x(x_0,y_0)\Delta x + F_y(x_0,y_0)\Delta y \\
&+ \tfrac{1}{2}F_{xx}(x_0,y_0)\Delta x^2 + F_{xy}(x_0,y_0)\Delta x\Delta y + \tfrac{1}{2}F_{yy}(x_0,y_0)\Delta y^2
\end{aligned}$$

Se $(x_0,y_0) = (0,0)$,

$$\begin{aligned}
z(x,y) &= z \\
&+ z_x x + z_y y \\
&+ \tfrac{1}{2}z_{xx}x^2 + z_{xy}xy + \tfrac{1}{2}z_{yy}y^2
\end{aligned}$$

$$\begin{aligned}
z(x_0+\Delta x, y_0+\Delta y) &= z \\
&+ z_x\Delta x + z_y\Delta y \\
&+ \tfrac{1}{2}z_{xx}\Delta x^2 + z_{xy}\Delta x\Delta y + \tfrac{1}{2}z_{yy}\Delta y^2
\end{aligned}$$

Questão 3
(Total: 5.0 pts)
Seja $H(x,y) = \sqrt{x^2 + 3y^2}$ e seja $(x_0, y_0) = (1, 1)$.
Encontre as aproximações de Taylor de ordem 1 e 2 para $H(x_0 + \Delta x, y_0 + \Delta y)$.

Questão 4
(Total: 1.0 pts)
Seja $M(x_0 + \Delta x, y_0 + \Delta y) = \Delta x(\Delta x + \Delta y)$.
Digamos que $(x_0, y_0) = (4, 3)$.
Faça o diagrama de numerozinhos da $M(x, y)$ nos pontos com $\Delta x, \Delta y \in \{-2, -1, -0, 1, 2\}$.

Questão 3: gabarito

Seja $H(x,y) = \sqrt{x^2+3y^2} = S$.
Então:

$$\begin{array}{rclrcl} H(x,y) &=& S & H(x_0,y_0) &=& 2 \\ H_x(x,y) &=& x/S & H_x(x_0,y_0) &=& 1/2 \\ H_y(x,y) &=& 3y/S & H_y(x_0,y_0) &=& 3/2 \\ H_{xx}(x,y) &=& (S^2-x^2)/S^3 & H_{xx}(x_0,y_0) &=& 3/8 \\ H_{xy}(x,y) &=& -3xy/S^3 & H_{xy}(x_0,y_0) &=& -3/8 \\ H_{yy}(x,y) &=& (3S^2-9y^2)/S^3 & H_{yy}(x_0,y_0) &=& 3/8 \end{array}$$

Aproximação de Taylor de 1a ordem:

$$\begin{array}{rcl} H(x_0+\Delta x, y_0+\Delta y) &\approx& H(x_0,y_0) \\ &+& H_x(x_0,y_0)\Delta x + H_y(x_0,y_0)\Delta y \\ &=& 2 \\ &+& \frac{1}{2}\Delta x + \frac{3}{2}\Delta y \end{array}$$

Aproximação de Taylor de 2a ordem:

$$\begin{array}{rcl} H(x_0+\Delta x, y_0+\Delta y) &\approx& H(x_0,y_0) \\ &+& H_x(x_0,y_0)\Delta x + H_y(x_0,y_0)\Delta y \\ &+& \frac{1}{2}H_{xx}(x_0,y_0)\Delta x^2 + H_{xy}(x_0,y_0)\Delta x\Delta y + \frac{1}{2}H_{yy}(x_0,y_0)\Delta y^2 \\ &=& 2 \\ &+& \frac{1}{2}\Delta x + \frac{3}{2}\Delta y \\ &+& \frac{3}{16}\Delta x^2 - \frac{3}{8}\Delta x\Delta y + \frac{3}{16}\Delta y^2 \end{array}$$

Questão 4: gabarito

0	-1	0	3	8
2	0	0	2	6
4	1	0	1	4
6	2	0	0	2
8	3	0	-1	0