Cálculo 2 - 2022.2

Aula 25: EDOs lineares
com coeficientes constantes

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http://angg.twu.net/2022.2-C2.html

Links:

EDOLCCs no “Diffy Qs” do Jiri Lebl:
https://www.jirka.org/diffyqs/diffyqs.pdf#page=84

EDOLCCs nas notas da Cristiane Hernández:
http://angg.twu.net/2015.1-C2/CALCULOIIA_EAD_Versao_Final_correcao_aulas_25_a_30.pdf#page=229

Questões sobre EDOLCCs nas provas de 2022.1:
http://angg.twu.net/LATEX/2022-1-C2-P2.pdf#page=5
http://angg.twu.net/LATEX/2022-1-C2-VR.pdf#page=4
http://angg.twu.net/LATEX/2022-1-C2-VSA.pdf#page=2

“Uma demonstração complicada:”
http://angg.twu.net/LATEX/2022-1-C2-der-fun-inv.pdf#page=5

Nessas aulas eu escrevi muitas coisas no quadro, e ainda não digitei elas. Você pode acessar as fotos dos quadros aqui:
http://angg.twu.net/2022.2-C2/C2-quadros.pdf#page=41

Material de 2019.2:
http://angg.twu.net/2019.2-C2/2019.2-C2.pdf#page=93
http://angg.twu.net/LATEX/2019-2-C2-tudo.pdf#page=10

Sobre o ‘$\cdot$’ em $42 \cdot f$ e em $f \cdot g$, veja os slides 15 até 21 daqui:
http://math.andrej.com/asset/data/the-dawn-of-formalized-mathematics.pdf
http://math.andrej.com/2021/06/24/the-dawn-of-formalized-mathematics/
https://vimeo.com/567049015

O Leithold define $f + g$, $f - g$ e $f \cdot g$ na página 36 (cap.1) e define a notação $\left.\frac{dy}{dx}\right]_{x=x_0}$ na p.145 (sec.3.1).

Capítulo sobre números complexos no “GA1” do Acker:
http://angg.twu.net/acker/acker__ga_livro1_2019.pdf#page=141

$$\begin{aligned}
(f+g)(x) &= f(x)+g(x) \\
f+g &= \lambda x.f(x)+g(x)
\end{aligned}$$

$$\begin{aligned}
(k\cdot f)(x) &= k\cdot f(x) \\
k\cdot f &= \lambda x.k\cdot f(x)
\end{aligned}$$

$$\begin{aligned}
(f\cdot g)(x) &= f(x)\cdot g(x) \\
f\cdot g &= \lambda x.f(x)\cdot g(x)
\end{aligned}$$

$$\begin{aligned}
(Df)(x) &= \tfrac{d}{dx}f(x) \\
Df &= \lambda x.\tfrac{d}{dx}f(x) \\
D &= \lambda f.\lambda x.\tfrac{d}{dx}f(x)
\end{aligned}$$

$$\begin{aligned}
D(g+h) &= (\lambda f.\lambda x.\tfrac{d}{dx}f(x))(g+h) \\
&= \lambda x.\tfrac{d}{dx}(g+h)(x) \\
&= \lambda x.\tfrac{d}{dx}(g(x)+h(x)) \\
&= \lambda x.\tfrac{d}{dx}g(x)+\tfrac{d}{dx}h(x)) \\
&= (\lambda x.\tfrac{d}{dx}g(x))+(\lambda x.\tfrac{d}{dx}h(x))) \\
&= Dg+Dh
\end{aligned}$$

$$\begin{aligned}
D(k\cdot g) &= (\lambda f.\lambda x.\tfrac{d}{dx}f(x))(k\cdot g) \\
&= \lambda x.\tfrac{d}{dx}(k\cdot g)(x) \\
&= \lambda x.\tfrac{d}{dx}(k\cdot g(x)) \\
&= \lambda x.k\cdot\tfrac{d}{dx}g(x) \\
&= k\cdot(\lambda x.\tfrac{d}{dx}g(x)) \\
&= k\cdot Dg
\end{aligned}$$

$$\begin{aligned}
D(\lambda x.x^2) &= (\lambda f.\lambda x.\tfrac{d}{dx}f(x))(\lambda x.x^2) \\
&= \lambda x.\tfrac{d}{dx}(\lambda x.x^2)(x) \\
&= \lambda x.\tfrac{d}{dx}x^2 \\
&= \lambda x.2x
\end{aligned}$$

$$\begin{aligned}
k\cdot f &= \lambda x.k\cdot f \\
k\cdot(\lambda x.e^{kx}) &= \lambda x.k\cdot(\lambda x.e^{kx})(x) \\
&= \lambda x.k\cdot e^{kx} \\
k\cdot(\lambda x.e^{kx}) &= \lambda x.k\cdot e^{kx} \\
42\cdot(\lambda x.e^{kx}) &= \lambda x.42e^{42x}
\end{aligned}$$

$$\begin{aligned}
D(\lambda x.e^{42x}) &= (\lambda f.\lambda x.\tfrac{d}{dx}f(x))(\lambda x.e^{42x}) \\
&= \lambda x.\tfrac{d}{dx}(\lambda x.e^{42x})(x) \\
&= \lambda x.\tfrac{d}{dx}e^{42x} \\
&= \lambda x.42e^{42x} \\
&= 42\cdot(\lambda x.e^{kx})
\end{aligned}$$

$$
\begin{aligned}
e^{i\theta} &= \cos\theta + i \operatorname{sen}\theta \\
\operatorname{sen} -\theta &= -\operatorname{sen}\theta \\
\cos -\theta &= -\cos\theta \\
e^{-i\theta} &= \cos -\theta + i \operatorname{sen} -\theta \\
&= \cos -\theta - i \operatorname{sen}\theta \\
&= \cos\theta - i \operatorname{sen}\theta \\
e^{i\theta} + e^{-i\theta} &= 2\cos\theta \\
e^{i\theta} - e^{-i\theta} &= 2i \operatorname{sen}\theta \\
\cos\theta &= \tfrac{1}{2}(e^{i\theta} + e^{-i\theta}) \\
\operatorname{sen}\theta &= \tfrac{1}{2i}(e^{i\theta} - e^{-i\theta}) \\
\cos k\theta &= \tfrac{1}{2}(e^{ik\theta} + e^{-ik\theta}) \\
\operatorname{sen} k\theta &= \tfrac{1}{2i}(e^{ik\theta} - e^{-ik\theta}) \\
e^{(\alpha+\beta i)\theta} &= e^{\alpha\theta} e^{\beta i\theta} \\
&= e^{\alpha\theta}(\cos\beta\theta + i \operatorname{sen}\beta\theta) \\
&= e^{\alpha\theta}\cos\beta\theta + i e^{\alpha}\operatorname{sen}\beta\theta \\
e^{(\alpha-\beta i)\theta} &= e^{\alpha\theta} e^{-\beta i\theta} \\
&= e^{\alpha\theta}(\cos(-\beta\theta) + i \operatorname{sen}(-\beta\theta)) \\
&= e^{\alpha\theta}(\cos\beta\theta - i \operatorname{sen}\beta\theta) \\
e^{(\alpha+\beta i)\theta} + e^{(\alpha-\beta i)\theta} &= e^{\alpha\theta}(e^{\beta i\theta} + e^{-\beta i\theta}) \\
&= 2\, e^{\alpha\theta}\cos\beta\theta \\
e^{(\alpha+\beta i)\theta} - e^{(\alpha-\beta i)\theta} &= e^{\alpha}(e^{\beta i\theta} - e^{-\beta i\theta}) \\
&= 2i\, e^{\alpha\theta}\operatorname{sen}\beta\theta \\
e^{\alpha}\cos\beta\theta &= \tfrac{1}{2}e^{(\alpha+\beta i)\theta} + \tfrac{1}{2}e^{(\alpha-\beta i)\theta} \\
e^{\alpha}\operatorname{sen}\beta\theta &= \tfrac{1}{2i}e^{(\alpha+\beta i)\theta} - \tfrac{1}{2i}e^{(\alpha-\beta i)\theta}
\end{aligned}
$$

$$
\begin{aligned}
E &= e^{i\theta} \\
c_\alpha &= \cos\alpha\theta \\
s_\alpha &= \operatorname{sen}\alpha\theta \\
E^\alpha &= (e^{i\theta})^\alpha \\
&= e^{i\alpha\theta} \\
&= c_\alpha + i s_\alpha \\
E^{-\alpha} &= e^{-i\alpha\theta} \\
&= c_{-\alpha} + i s_{-\alpha} \\
&= c_\alpha - i s_\alpha \\
c_\alpha &= \tfrac{1}{2}(E^\alpha + E^{-\alpha}) \\
s_\alpha &= \tfrac{1}{2i}(E^\alpha - E^{-\alpha}) \\
E^{\alpha+i\beta} &= \\
e^{(\alpha+\beta i)\theta} + e^{(\alpha-\beta i)\theta} &= e^{\alpha}(e^{\beta i\theta} + e^{-\beta i\theta}) \\
&= 2\, e^{\alpha}\cos\beta\theta \\
e^{(\alpha+\beta i)\theta} - e^{(\alpha-\beta i)\theta} &= e^{\alpha}(e^{\beta i\theta} - e^{-\beta i\theta}) \\
&= 2i\, e^{\alpha}\operatorname{sen}\beta\theta \\
e^{\alpha}\cos\beta\theta &= \tfrac{1}{2}e^{(\alpha+\beta i)\theta} + \tfrac{1}{2}e^{(\alpha-\beta i)\theta} \\
e^{\alpha}\operatorname{sen}\beta\theta &= \tfrac{1}{2i}e^{(\alpha+\beta i)\theta} - \tfrac{1}{2i}e^{(\alpha-\beta i)\theta}
\end{aligned}
$$