Cálculo C2 - 2023.2

Aula 30: EDOs lineares
com coeficientes constantes

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http://anggtwu.net/2023.2-C2.html

Links

StewPtCap17p5 (p.1020) Equações diferenciais de 2ª ordem
StewPtApendiceHp5 (p.A51) Apêndice H: Números complexos

Leit3p22 (p.158) $D_x[c \cdot f(x)] = c \cdot D_x f(x)$
Leit3p22 (p.158) $D_x[f(x) + g(x)] = D_x f(x) + D_x g(x)$

BoyceDip3p5 (p.105) Capítulo 3: Equações lineares de 2ª ordem
BoyceDip3p11 (p.111) Seção 3.2: o operador diferencial L
BoyceDip3p13 (p.113) Teorema 3.2.2: o princípio da superposição

ZillCullenCap4p33 (p.173) 4.3. Equações lineares homogêneas com coeficientes constantes

BoyceDipEng3p4 (p.103) Chapter 3: Second-order linear ODEs
BoyceDipEng3p11 (p.110) Section 3.2: the differential operator L
BoyceDipEng3p13 (p.112) Theorem 3.2.2: principle of superposition

Quadros:
2hQ61 Aula 30 de 2023.2 (01/nov/2023)
Aviso: falta digitar muita coisa! Veja os quadros!

Quadros antigos:
2gQ46 Aula 23 de 2023.1 (20/junho/2023)
2gQ50 Aula 24 de 2023.1 (23/junho/2023)

$$\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&0\\1&0\end{pmatrix}=\begin{pmatrix}1&0\\0&0\end{pmatrix}$$
$$\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&1\end{pmatrix}$$

$$\begin{pmatrix}a&b\end{pmatrix}\begin{pmatrix}c\\d\end{pmatrix}=\begin{pmatrix}ac+bd\end{pmatrix}$$
$$\begin{pmatrix}c\\d\end{pmatrix}\begin{pmatrix}a&b\end{pmatrix}=\begin{pmatrix}ac&bc\\ad&bd\end{pmatrix}$$

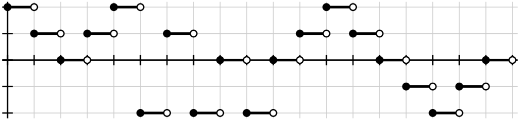
$$S=\begin{pmatrix}0&1&&&\\&0&1&&\\&&0&1&\\&&&0&1\\&&&&0\end{pmatrix}\quad 1=\begin{pmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&1\end{pmatrix}\quad S-1=\begin{pmatrix}-1&1&&&\\&-1&1&&\\&&-1&1&\\&&&-1&1\\&&&&-1\end{pmatrix}$$

$$v=\begin{pmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{pmatrix}\quad f=\begin{pmatrix}f(1)\\f(2)\\f(3)\\f(4)\\f(5)\end{pmatrix}\quad Sf=\begin{pmatrix}f(2)\\f(3)\\f(4)\\f(5)\\0\end{pmatrix}\quad (S-1)f=\begin{pmatrix}f(2)-f(1)\\f(3)-f(2)\\f(4)-f(3)\\f(5)-f(4)\\0-f(5)\end{pmatrix}$$

Obs: não usei isso aqui –
não deu tempo de LaTeXar tudo...

$g(x) \quad =$

$g'(x) \quad =$



```
(%i1) f   : exp( 3*x);
(%o1)
```
$$e^{3x}$$
```
(%i2) f   : exp(-3*x);
(%o2)
```
$$e^{-3x}$$
```
(%i3) f   : exp( 2*x);
(%o3)
```
$$e^{2x}$$
```
(%i4) fp  : diff(f,x);
(%o4)
```
$$2\,e^{2x}$$
```
(%i5) fpp : diff(f,x,2);
(%o5)
```
$$4\,e^{2x}$$
```
(%i6) Lf  : fpp + fp - 6*f;
(%o6)
```
$$0$$
```
(%i7)
```

```
(%i1) D (f) := diff(f,x);
(%o1)
```
$$D\,(f) := \mathrm{diff}\,(f, x)$$
```
(%i2) DD(f) := diff(f,x,2);
(%o2)
```
$$\mathrm{DD}\,(f) := \mathrm{diff}\,(f, x, 2)$$
```
(%i3) L (f) := D(D(f)) + D(f) - 6*f;
(%o3)
```
$$L\,(f) := D\,(D\,(f)) + D\,(f) + (-6)\;f$$
```
(%i4)   D(x^2);
(%o4)
```
$$2\,x$$
```
(%i5) D(D(x^2));
(%o5)
```
$$2$$
```
(%i6) L(x^2);
(%o6)
```
$$-\left(6\,x^2\right) + 2\,x + 2$$
```
(%i7) L(exp( 3*x));
(%o7)
```
$$6\,e^{3x}$$
```
(%i8) L(exp(-3*x));
(%o8)
```
$$0$$
```
(%i9) L(exp( 2*x));
(%o9)
```
$$0$$
```
(%i10)
```

Soluções não-básicas

$$\begin{aligned} M(\alpha v + \beta w) &= M(\alpha v) + M(\beta w) \\ &= \alpha(Mv) + \beta(Mw) \end{aligned}$$

$$\underbrace{(D-2)(D+3)}_{M}(\underbrace{42}_{\alpha}\underbrace{e^{2x}}_{v} + \underbrace{99}_{\beta}\underbrace{e^{-3x}}_{w})$$

$$\begin{aligned} &(D-2)(D+3)(42e^{2x} + 99e^{-3x}) \\ &= 42(D-2)(D+3)e^{2x} + 99(D-2)(D+3)e^{-3x} \\ &= \underbrace{42\,\underbrace{(D+3)\,\underbrace{\underbrace{\underbrace{(D-2)e^{2x}}_{De^{2x}-2e^{2x}}}_{2e^{2x}-2e^{2x}}}_{0}}_{0}}_{0} + \underbrace{99\,\underbrace{(D-2)\,\underbrace{\underbrace{\underbrace{(D+3)e^{-3x}}_{De^{-3x}+3e^{-3x}}}_{-3e^{-3x}+3e^{-3x}}}_{0}}_{0}}_{0} \end{aligned}$$