Cálculo 3 - 2023.2

Aula 26: matrizes definidas

Eduardo Ochs - RCN/PURO/UFF
http://anggtwu.net/2023.2-C3.html

Links

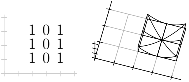
Strang4cap3p42 (p.159) 3 Orthogonality
Strang4cap3p42 (p.196) Orthogonal Matrices
Strang4cap5p5 (p.260) 5 Eigenvalues and Eigenvectors
Strang4cap5p12 (p.267) Eigshow
Strang4cap5p42 (p.312) Every symmetric matrix ... has real eigenvalues
Strang4cap5p42 (p.312) Its eigenvectors can be chosen to be orthonormal
Strang4cap6p5 (p.345) 6 Positive Definite Matrices
Strang4cap6p15 (p.355) Cholesky decomposition

Bort11p15 (p.379) O polinômio de Taylor de ordem 2
Bort11p16 (p.380) matriz Hessiana
Bort11p19 (p.383) 11.3 Formas quadráticas e matrizes definidas

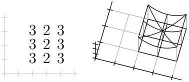
StewPtCap14p64 (p.850) 14.7 Valores Máximo e Mínimo
StewPtCap14p65 (p.851) Teste da segunda derivada; D(a,b)

Exemplos com numerozinhos

Δx^2



$2 + \Delta x^2$



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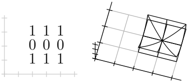

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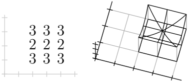
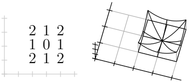
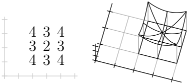


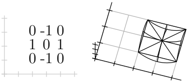
$\Delta x^2 + \Delta y^2$



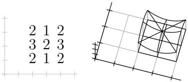
$2 + \Delta x^2 + \Delta y^2$



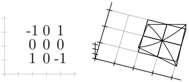
$\Delta x^2 - \Delta y^2$



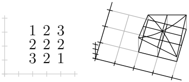
$2 + \Delta x^2 - \Delta y^2$



$\Delta x \cdot \Delta y$



$2 + \Delta x \cdot \Delta y$



Versão mega-rápida das páginas 365–394 do Bortolossi

Links:
http://angg.twu.net/LATEX/2022-2-C3-funcoes-homogeneas.pdf
http://angg.twu.net/2022.2-C3/C3-quadros.pdf#page=17

Digamos que:

$$\begin{array}{rcl} r_1, r_2, r \in \mathbb{R}, & & r_1 \neq r_2, \\ \alpha \in \mathbb{R}, & & \alpha > 0, \\ \beta, \gamma \in \mathbb{R}, & & \gamma > 0, \\ r_3 = \beta + i\gamma, & & r_4 = \beta - i\gamma, \\ z(x,y) & = & dx^2 + exy + fy^2, \\ h(x) & = & z(x,1). \end{array}$$

Então:

se	$h(x) = (x - r_1)(x - r_2)$	então $(0,0)$ é um ponto de sela,
se	$h(x) = \alpha(x - r_1)(x - r_2)$	então $(0,0)$ é um ponto de sela,
se	$h(x) = -\alpha(x - r_1)(x - r_2)$	então $(0,0)$ é um ponto de sela,
se	$h(x) = (x - r)^2$	então $(0,0)$ é $z = x^2$,
se	$h(x) = \alpha(x - r)^2$	então $(0,0)$ é $z = x^2$,
se	$h(x) = -\alpha(x - r)^2$	então $(0,0)$ é $z = -x^2$,
se	$h(x) = (x - r_3)(x - r_4)$	então $(0,0)$ "tem concavidade pra cima",
se	$h(x) = \alpha(x - r_3)(x - r_4)$	então $(0,0)$ "tem concavidade pra cima",
se	$h(x) = -\alpha(x - r_3)(x - r_4)$	então $(0,0)$ "tem concavidade pra baixo".

Se $z = z(x,y) = ax^2 + bxy + cy^2$

e $M = \begin{pmatrix} z_{xx} & z_{xy} \\ z_{xy} & z_{yy} \end{pmatrix}$

então $z_{xx} = 2a$

$z_{xy} = b$

$z_{yy} = 2c$

$M = \begin{pmatrix} 2a & b \\ b & 2c \end{pmatrix}$

Se $z = z(x,y)$

$x = x(t)$

$y = y(t)$

$x_{tt} = 0$

$y_{tt} = 0$

então $z_{tt} = z_{xx}x_t x_y + 2z_{xy}x_t y_t + z_{yy}y_t y_t$

e $z_{tt} = \begin{pmatrix} x_t \\ y_t \end{pmatrix}^T \begin{pmatrix} z_{xx} & z_{xy} \\ z_{xy} & z_{yy} \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix}$

Se $c = \cos\theta$

$s = \operatorname{sen}\theta$

$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$

$x_1 = \begin{pmatrix} c \\ s \end{pmatrix}$

$x_2 = \begin{pmatrix} -s \\ c \end{pmatrix}$

$S = \begin{pmatrix} | & | \\ x_1 & x_2 \\ | & | \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$

$A = S\Lambda S^{-1}$

então $S^{-1} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$

$S\begin{pmatrix} 1 \\ 0 \end{pmatrix} = x_1, \quad S^{-1}x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$S\begin{pmatrix} 0 \\ 1 \end{pmatrix} = x_2, \quad S^{-1}x_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$Ax_1 = S\Lambda S^{-1}x_1 = \lambda_1 x_1$

$Ax_2 = S\Lambda S^{-1}x_2 = \lambda_2 x_2$

$x_1^T A x_1 = \lambda_1$

$x_2^T A x_2 = \lambda_2$

Se além disso

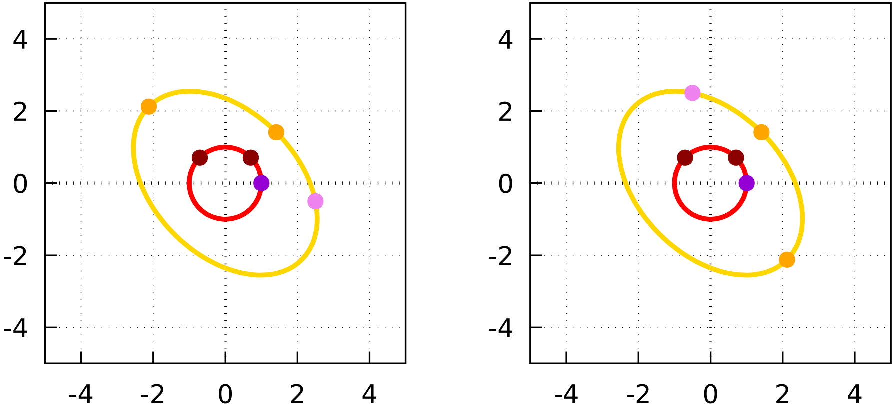
tudo temos $M = A = S\Lambda S^1$

então quando $\begin{pmatrix} x_t \\ y_t \end{pmatrix} = x_1 = \begin{pmatrix} c \\ s \end{pmatrix}$

temos $z_{tt} = \lambda_1$

e quando $\begin{pmatrix} x_t \\ y_t \end{pmatrix} = x_2 = \begin{pmatrix} -s \\ c \end{pmatrix}$

temos $z_{tt} = \lambda_2$.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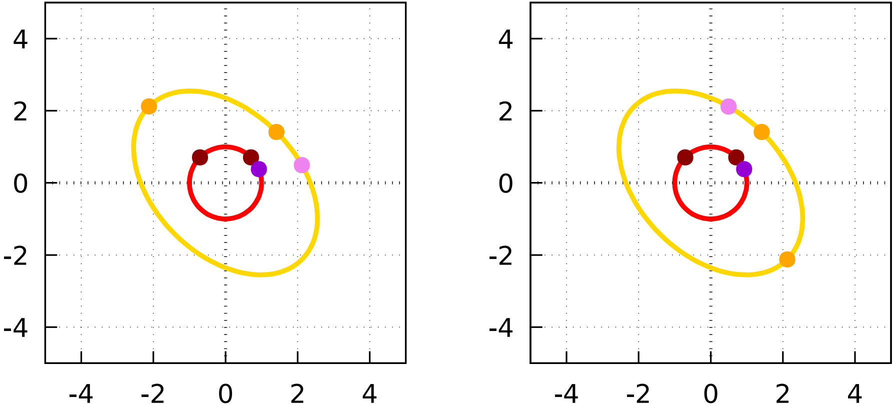
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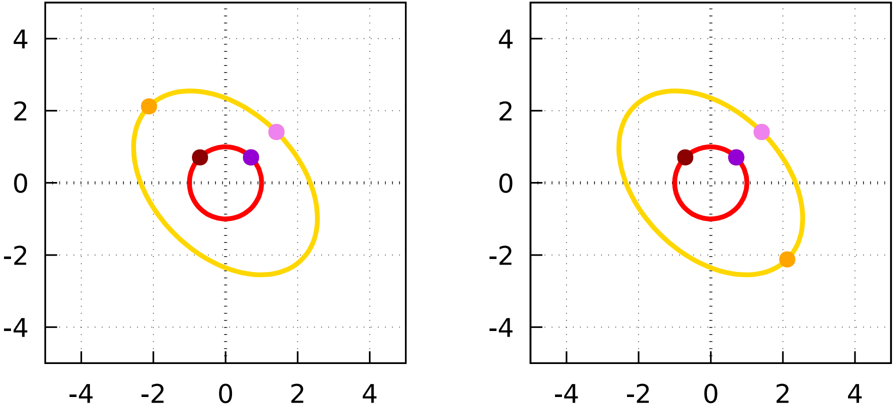
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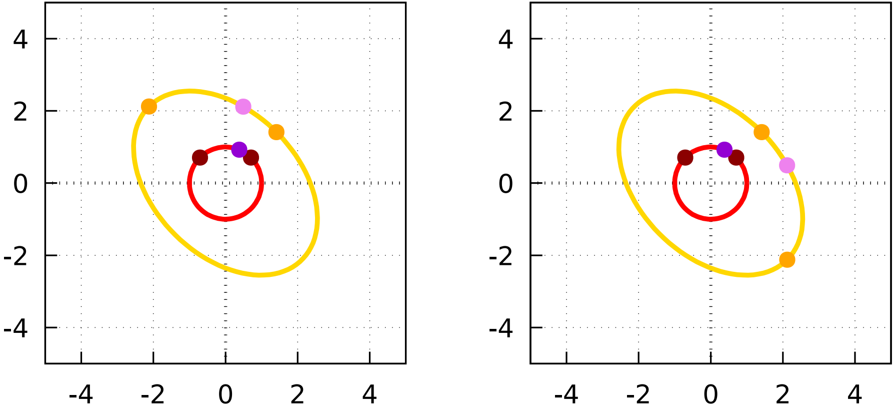

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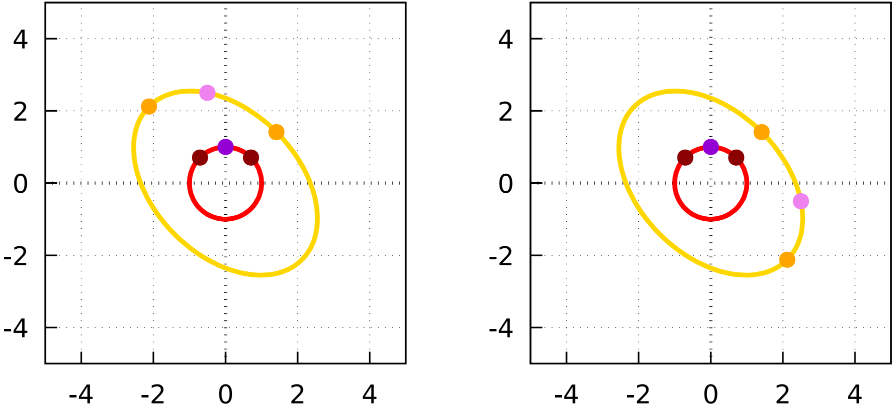
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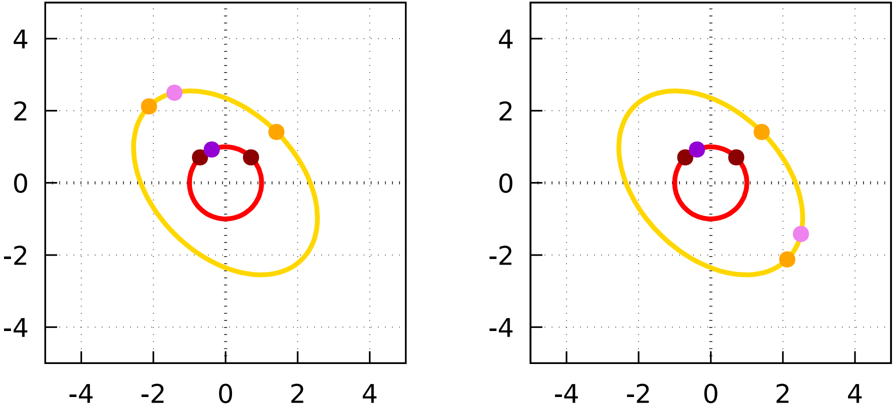
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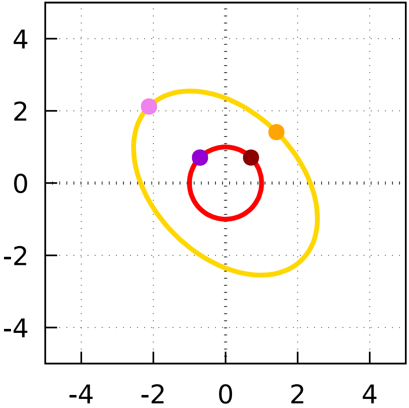
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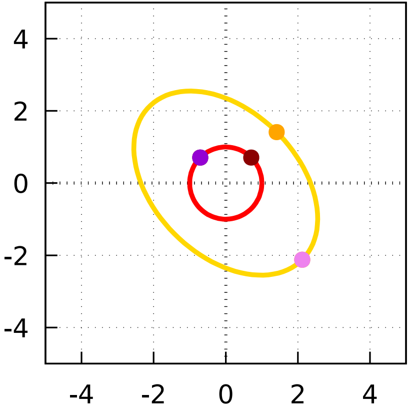
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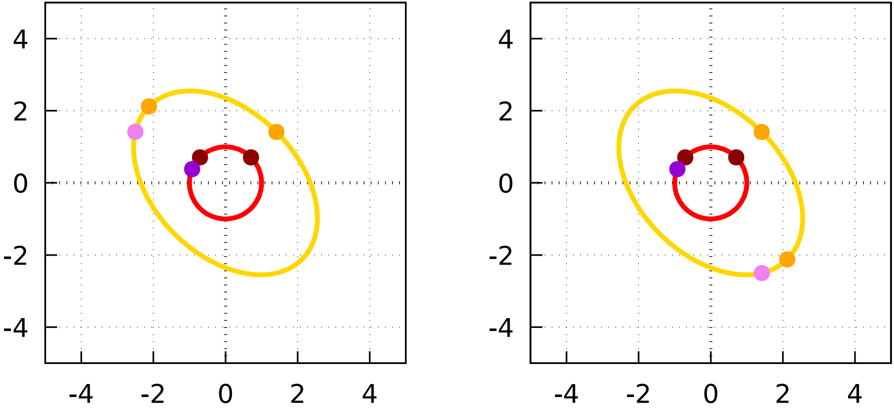

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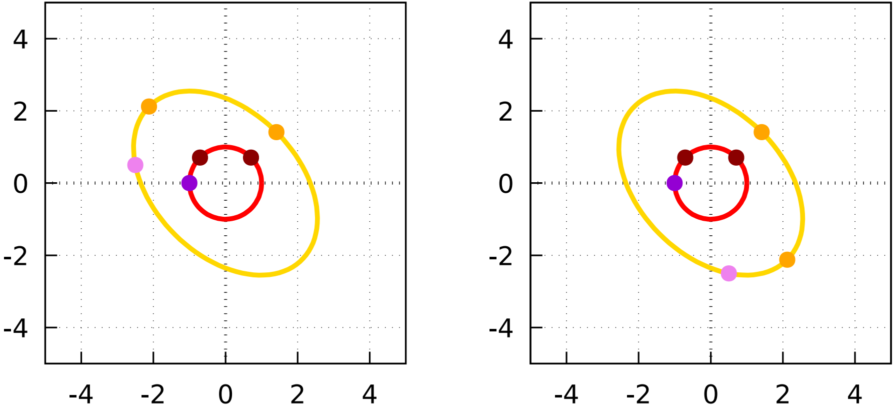
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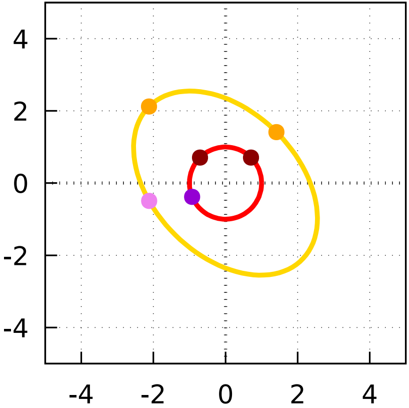
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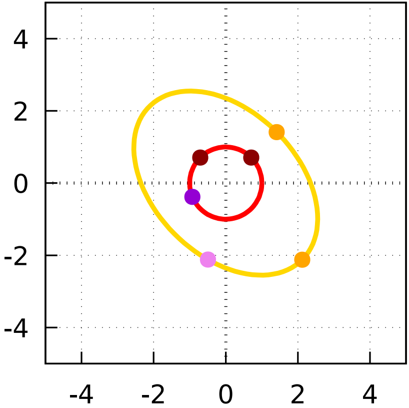
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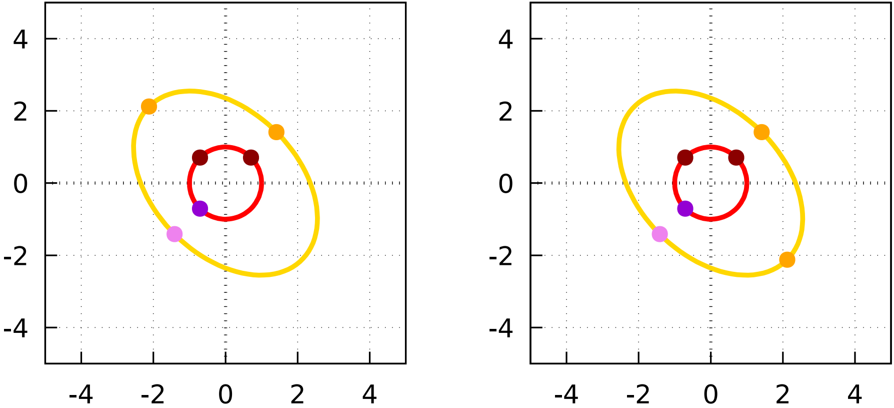
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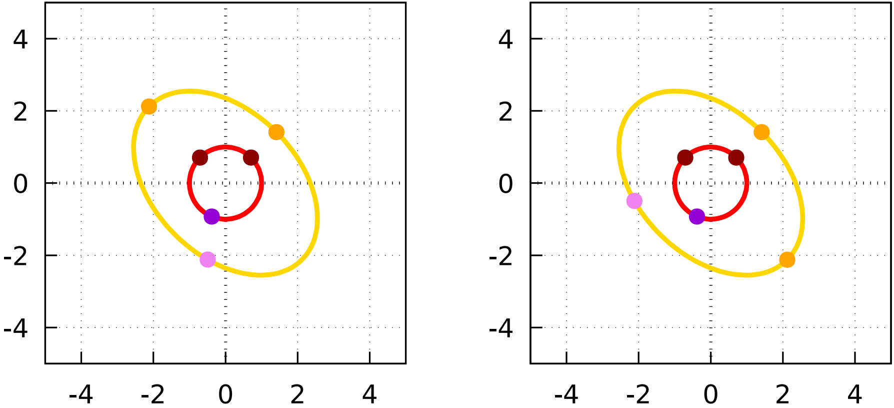
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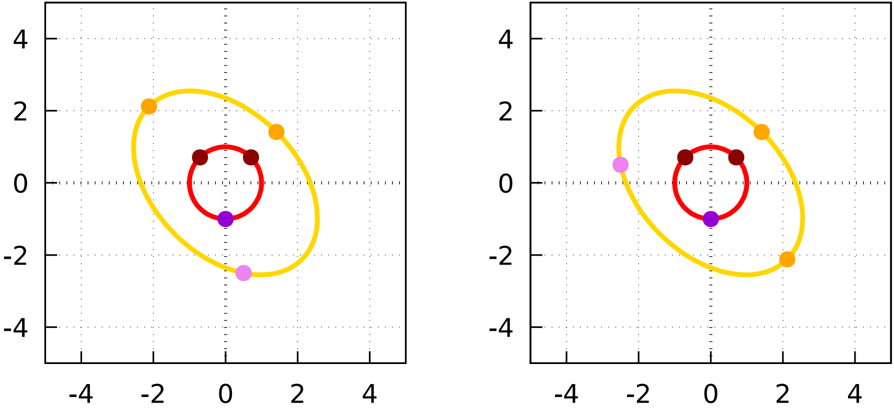
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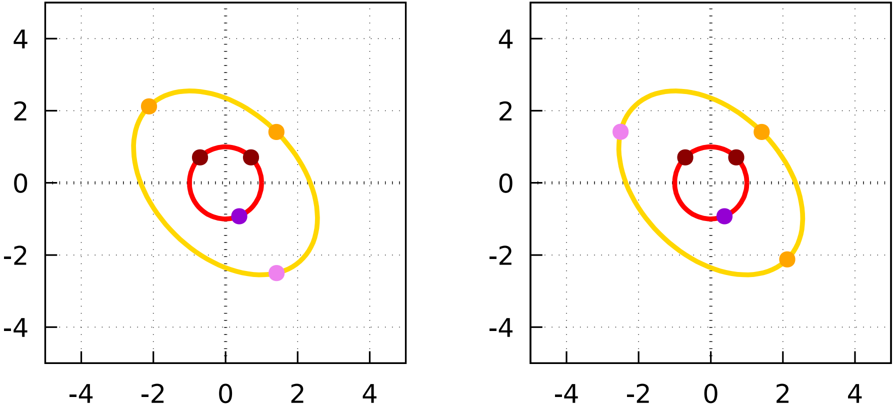
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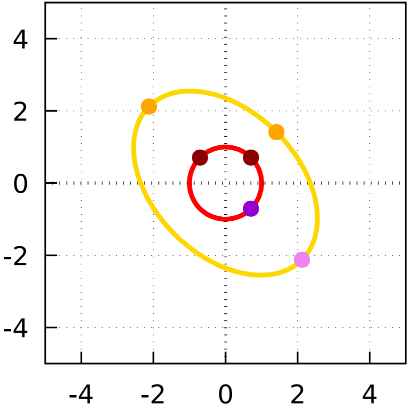
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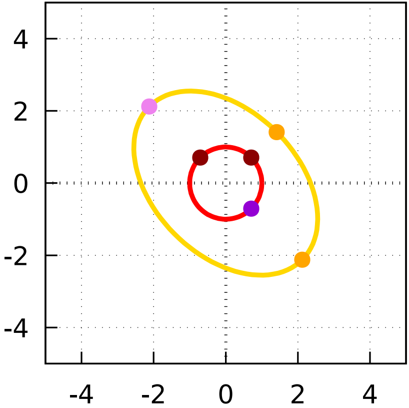
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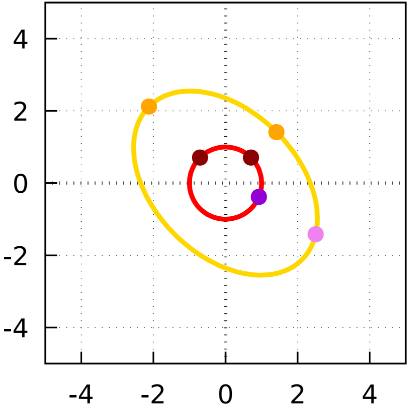
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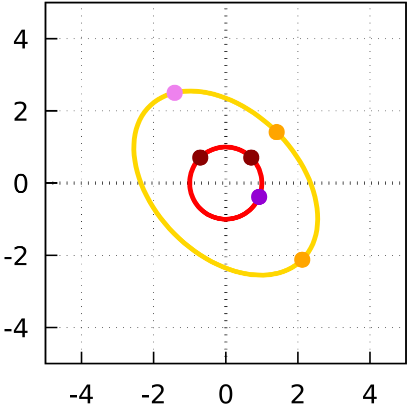
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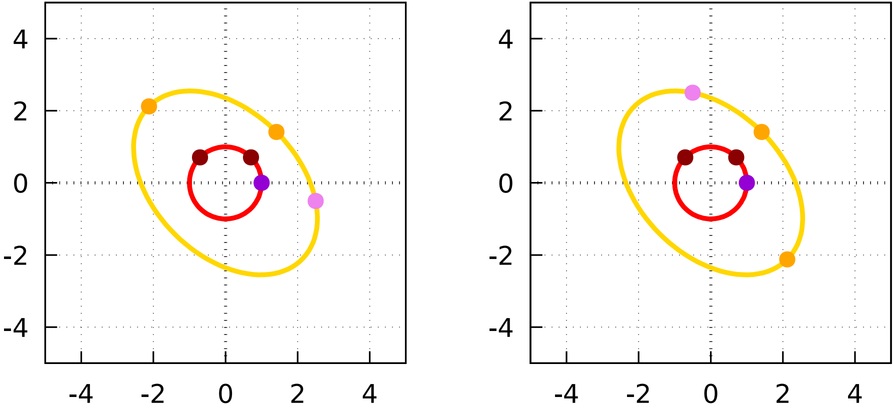
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