Cálculo 3 - 2024.1

Aula 21: A diferencial total

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http://anggtwu.net/2024.1-C3.html

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StewPtCap14p41 (p.827) a diferencial dz ... diferenciação total
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(find-es "maxima" "2024.1-depends")
(find-es "maxima" "2024.1-gradefs")

Depends

```
(%i1) y : x^2;
```
(%o1) x^2

```
(%i2) z : sin(y);
```
(%o2) $\sin x^2$

```
(%i3) diff(z, x);
```
(%o3) $2\,x\,\cos x^2$

```
(%i4) diff(z, x, 2);
```
(%o4) $2\cos x^2 - 4\,x^2\,\sin x^2$

```
(%i5) values;
```
(%o5) $[y, z]$

```
(%i6)
     remvalue(all);
```
(%o6) $[y, z]$

```
(%i7) values;
```
(%o7) []

```
(%i8) diff(y, x);
```
(%o8) 0

```
(%i9) depends(y, x);
```
(%o9) $[y\,(x)]$

```
(%i10) depends(z, y);
```
(%o10) $[z\,(y)]$

```
(%i11) dependencies;
```
(%o11) $[y\,(x)\,,z\,(y)]$

```
(%i12) diff(y, x);
```
(%o12) $\frac{d}{dx}\,y$

```
(%i13) diff(z, x);
```
(%o13) $\frac{d}{dx}\,y\left(\frac{d}{dy}\,z\right)$

```
(%i14) diff(z, x, 2);
```
(%o14) $\left(\frac{d}{dx}\,y\right)^2\left(\frac{d^2}{dy^2}\,z\right)+\frac{d^2}{dx^2}\,y\left(\frac{d}{dy}\,z\right)$

```
(%i15) o1 : diff(z, x);
```
(%o15) $\frac{d}{dx}\,y\left(\frac{d}{dy}\,z\right)$

```
(%i16) o2 : diff(z, x, 2);
```
(%o16) $\left(\frac{d}{dx}\,y\right)^2\left(\frac{d^2}{dy^2}\,z\right)+\frac{d^2}{dx^2}\,y\left(\frac{d}{dy}\,z\right)$

```
(%i17) o3 : at(o1, [z=sin(y), y=x^2]);
```
(%o17) $\frac{d}{dx}\,x^2\left(\frac{d}{dy}\,\sin y\Big|_{y=x^2}\right)$

```
(%i18) o4 : at(o2, [z=sin(y), y=x^2]);
```
(%o18) $\left(\frac{d}{dx}\,x^2\right)^2\left(\frac{d^2}{dy^2}\,\sin y\Big|_{y=x^2}\right)+\frac{d^2}{dx^2}\,x^2\left(\frac{d}{dy}\,\sin y\Big|_{y=x^2}\right)$

```
(%i19) o5 : ev(o3, 'derivative, 'at);
```
(%o19) $2\,x\,\cos x^2$

```
(%i20) o6 : ev(o4, 'derivative, 'at);
```
(%o20) $2\cos x^2 - 4\,x^2\,\sin x^2$

```
(%i21)
```

Gradefs

```
(%i1) diff  (sin(x), x);
(%o1)
```

$$\cos x$$

```
(%i2) gradef(sin(x), sqrt(1-sin(x)^2));
(%o2)
```

$$\sin x$$

```
(%i3) diff  (sin(x), x);
(%o3)
```

$$\sqrt{1-(\sin x)^2}$$

```
(%i4)  texput(y_x,  "y_x")$
(%i5)  texput(y_xx, "y_{xx}")$
(%i6)  texput(z_y,  "z_y")$
(%i7)  texput(z_yy, "z_{yy}")$
(%i8)  gradef(y,x,   y_x)$
(%i9)  gradef(z,y,   z_y)$
(%i10) gradef(y_x,x, y_xx)$
(%i11) gradef(z_y,y, z_yy)$
```

```
(%i12) diff(y, x);
(%o12)
```

$$y_x$$

```
(%i13) diff(z, x);
(%o13)
```

$$y_x\, z_y$$

```
(%i14) diff(z, x, 2);
(%o14)
```

$$y_x^2\, z_{yy} + y_{xx}\, z_y$$

```
(%i15) o1 : diff(y);
(%o15)
```

$$y_x\, dx$$

```
(%i16) o2 : diff(z);
(%o16)
```

$$z_y\, dy$$

```
(%i17) o3 : subst([del(y)=diff(y)], o2);
(%o17)
```

$$y_x\, z_y\, dx$$

```
(%i18)
```