Cálculo 2 - 2024.2

Aulas 23 a 26: revisão de números complexos

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http://anggtwu.net/2024.2-C2.html

Links

Quadros:
2jQ55 (2024.2)
2iQ81 (2024.1)
2hQ66 (2023.2)
2fQ45 (2022.2)
2yQ63 (2019.2)

Provas:
2yT12 (Gabarito da P1 de 2019.2) A questão 3 usa o truque do E
http://anggtwu.net/LATEX/2018-2-C2-P1.pdf#page=2 Questão 1

StewPtCap17p6 (p.1020) Equações diferenciais de 2ª ordem
StewPtCap17p20 (p.1034) Caso 3: subamortecimento
StewPtApendiceHp5 (p.A51) Apêndice H: Números complexos

BoyceDip3p5 (p.105) Capítulo 3: Equações lineares de 2ª ordem
BoyceDip3p11 (p.111) Seção 3.2: o operador diferencial L
BoyceDip3p13 (p.113) Teorema 3.2.2: o princípio da superposição
BoyceDip3p21 (p.121) 3.3. Raízes complexas da equação característica
BoyceDip3p23 (p.123) Figura 3.3.1

BoyceDipEng3p4 (p.103) Chapter 3: Second-order linear ODEs
BoyceDipEng3p11 (p.110) Section 3.2: the differential operator L
BoyceDipEng3p13 (p.112) Theorem 3.2.2: principle of superposition
BoyceDipEng3p21 (p.120) 3.3 Complex Roots of the Characteristic Equation
BoyceDipEng3p24 (p.123) Figure 3.3.1

http://www.youtube.com/watch?v=-dhHrg-KbJ0 $e^{\pi i}$ for dummies (Mathologer)
https://en.wikipedia.org/wiki/Complex_number (bom)
https://pt.wikipedia.org/wiki/N%C3%BAmero_complexo (ruim, cheio de erros)

HernandezP57 (p.47) principais identidades trigonométricas

```
(%i1) Re(z)    := realpart(z)$
(%i2) Im(z)    := imagpart(z)$
(%i3) sqhyp(z) := Re(z)^2 + Im(z)^2$
(%i4) tom(z)   := matrix([Re(z),-Im(z)], [Im(z),Re(z)])$
(%i5) det(M)   := determinant(M)$
(%i6) nm(z)    := Re(z) + %i*Im(z)$
(%i7) stringdisp : false$
(%i8) z : a + %i*b;
```

(%o8)

$$i\,b + a$$

```
(%i9) w : c + %i*d;
```

(%o9)

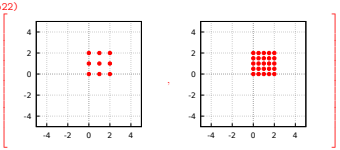
$$i\,d + c$$

```
(%i10) matrix([    z , "+",     w , "=",   nm(z+w)],
               [tom(z), "+", tom(w), "=", tom(z)+tom(w)],
               [    "",  "",     "",  "",       ""     ],
               [    z , "*",     w , "=",   nm(z*w)    ],
               [tom(z), ".", tom(w), "=", tom(z).tom(w)]);
```

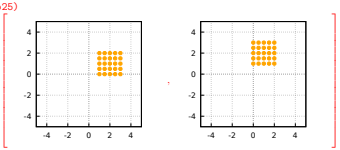
(%o10)

$$\begin{pmatrix} i\,b+a & + & i\,d+c & = & i\,(d+b)+c+a \\ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} & + & \begin{pmatrix} c & -d \\ d & c \end{pmatrix} & = & \begin{pmatrix} c+a & -d-b \\ d+b & c+a \end{pmatrix} \\ & & & & \\ i\,b+a & * & i\,d+c & = & i\,(a\,d+b\,c)-b\,d+a\,c \\ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} & . & \begin{pmatrix} c & -d \\ d & c \end{pmatrix} & = & \begin{pmatrix} a\,c-b\,d & -(a\,d)-b\,c \\ a\,d+b\,c & a\,c-b\,d \end{pmatrix} \end{pmatrix}$$

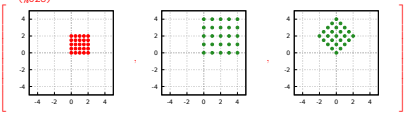
```
(%i11)  drawzpts (zs,[opts]) := myqdrawp(xyrange(), zpts(zs, opts))$
(%i12) [xmin,ymin, xmax,ymax] :  [-5,-5, 5,5]$
(%i13) myqdrawp_to_screen()$  myps(size) := ps(size)$
(%i15) myqdrawp_to_new_pdf()$ myps(size) := ps(size/5)$
(%i17)  as_33     : create_list(x+%i*y, y,seqn(2,0,2), x,seqn(0,2,2));
(%o17)
                    [2 i, 2 i + 1, 2 i + 2, i, i + 1, i + 2, 0, 1, 2]

(%i18) as_55      : create_list(x+%i*y, y,seqn(2,0,4), x,seqn(0,2,4))$
(%i19) as_22      : create_list(x+%i*y, y,seqn(0,1,1), x,seqn(0,1,1))$
(%i20)  D1 : drawzpts(as_33,          myps(3), pc(red))$
        D2 : drawzpts(as_55,          myps(3), pc(red))$

(%i21)
(%i22) [D1, D2];
(%o22)
```




```
(%i23)  D3 : drawzpts(as_55 + 1,      myps(3), pc(orange))$

(%i24)  D4 : drawzpts(as_55 + %i,     myps(3), pc(orange))$

(%i25) [D3, D4];
(%o25)
```



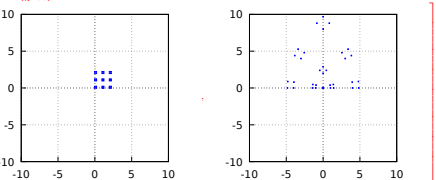
```
(%i26)  D5 : drawzpts(as_55 * 2,      myps(3), pc(forest_green))$
(%i27)  D6 : drawzpts(as_55 * (1+%i), myps(3), pc(forest_green))$
(%i28) [D2, D5, D6];
(%o28)
```



```
(%i29) topdf_opts : "height=10cm"$
(%i30) as_332    : create_list(z+w, z,as_33, w,as_22*0.2)$
(%i31) as_332_sq : makelist(z^2, z, as_332)$
(%i32) [xmin,ymin, xmax,ymax] :  [-10,-10, 10,10];
(%o32)
```

$$[-10,-10,10,10]$$

```
(%i33)  D7 : drawzpts(as_332,    myps(0.5))$
(%i34)  D8 : drawzpts(as_332_sq, myps(0.5))$
(%i35) [D7, D8];
(%o35)
```



```
(%i36)
```

2019.2

$$
\begin{aligned}
&(\operatorname{sen} 5\theta)^2(\cos 6\theta)^2 \\
&= \left(\tfrac{E^5-E^{-5}}{2i}\right)^2\left(\tfrac{E^6+E^{-6}}{2}\right)^2 \\
&= -\tfrac{1}{16}(E^{10}-2+E^{-10})(E^{12}+2+E^{-12}) \\
&= -\tfrac{1}{16}\begin{pmatrix}(E^{10}-2+E^{-10})E^{12}+\\ (E^{10}-2+E^{-10})\cdot 2+\\ (E^{10}-2+E^{-10})E^{-12}\end{pmatrix} \\
&= -\tfrac{1}{16}\begin{pmatrix}E^{22}-2E^{12}+E^2+\\ 2E^{10}-4+2E^{-10}+\\ E^{-2}-2E^{-12}+E^{-22}\end{pmatrix} \\
&= -\tfrac{1}{16}((E^{22}+E^{-22})-2(E^{12}+E^{-12})+2(E^{10}+E^{-10})+(E^2+E^{-2})-4) \\
&= -\tfrac{1}{16}(2\cos 22\theta-4\cos 12\theta+4\cos 10\theta+2\cos 2\theta-4) \\
&= -\tfrac{1}{8}\cos 22\theta+\tfrac{1}{4}\cos 12\theta-\tfrac{1}{4}\cos 10\theta-\tfrac{1}{8}\cos 2\theta\tfrac{1}{4}
\end{aligned}
$$

$$
\begin{aligned}
\int(\operatorname{sen} 5x)^2(\cos 6x)^2\,dx &= \int -\tfrac{1}{8}\cos 22x+\tfrac{1}{4}\cos 12x-\tfrac{1}{4}\cos 10x-\tfrac{1}{8}\cos 2x+\tfrac{1}{4}\,dx \\
&= -\tfrac{1}{8\cdot 22}\operatorname{sen} 22x+\tfrac{1}{4\cdot 12}\operatorname{sen} 12x-\tfrac{1}{4\cdot 10}\operatorname{sen} 10x-\tfrac{1}{8\cdot 2}\operatorname{sen} 2x+\tfrac{1}{4}x
\end{aligned}
$$