

Emacs, eev, and Maxima – Now!

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<http://anggtwu.net/emacsconf2024.html>

<https://emacsconf.org/2024/talks/maxima/>

Links

<http://anggtwu.net/math-b.html#2022-md>

<http://www.math.jhu.edu/~eriehl/>

<http://www.math.jhu.edu/~eriehl/context.pdf>

Yoneda on rings

This is the example (iv) in p.52 of Emily Riehl's "*Category Theory in Context*":

The functor $U : \mathbf{Ring} \rightarrow \mathbf{Set}$ is represented by the unital ring $\mathbb{Z}[x]$, the polynomial ring in one variable with integer coefficients. A unital ring homomorphism $\mathbb{Z}[x] \rightarrow R$ is uniquely determined by the image of x ; put another way, $\mathbb{Z}[x]$ is the *free unital ring on a single generator*.

$$\begin{array}{ccccc}
 FA \longleftarrow A & & A & & 1 \\
 \downarrow & \longleftarrow & \downarrow \eta_A & & \downarrow \eta_1 \\
 B \longrightarrow UB & & UFA & & U\mathbb{Z}[x] \\
 \mathbf{Ring} \xrightleftharpoons[U]{F} \mathbf{Set} & & & & \mathbb{Z}[x] \longmapsto U\mathbb{Z}[x] \\
 & & & & \mathbf{Ring} \xrightarrow{U} \mathbf{Set} \\
 & & & & \mathbf{Ring}(\mathbb{Z}[x], -) \\
 & & & & \swarrow \quad \searrow U
 \end{array}$$

Yoneda on rings

$$\begin{array}{ccc}
 FA \longleftarrow \dashv A & A & 1 \\
 \downarrow & \downarrow \eta_A & \downarrow \eta_1 \\
 B \dashv \longrightarrow UB & UFA & U\mathbb{Z}[x] \\
 & & \downarrow \text{univ} \\
 & & \mathbb{Z}[x] \dashv \longrightarrow U\mathbb{Z}[x]
 \end{array}$$

$$\text{Ring} \begin{array}{c} \xleftarrow{F} \\ \xrightarrow{U} \end{array} \text{Set}$$

$$\text{Ring} \xrightarrow{U} \text{Set}$$

$$\text{Ring}(\mathbb{Z}[x], -) \begin{array}{c} \swarrow \\ \searrow \end{array} U$$

$$\begin{array}{ccc}
 & & A \\
 & & \downarrow \eta \\
 C & \xrightarrow{\quad} & RC \\
 & \nearrow & \\
 B & \xrightarrow{R} & A \\
 & \searrow & \\
 B(C, -) & \xrightarrow{\alpha} & A(A, R-)
 \end{array}$$

\mathbf{A} is a category,

\mathbf{B} is a category,

$R : \mathbf{B} \rightarrow \mathbf{A}$,

$A \in \mathbf{A}$,

$C \in \mathbf{B}$,

$\eta : A \rightarrow RC$,

$\mathbf{B}(C, -) : \mathbf{B} \rightarrow \mathbf{Set}$,

$\mathbf{B}(C, -)_0 := \lambda D. \mathbf{B}(C, D)$,

$\mathbf{B}(C, -)_1 := \lambda g. \lambda f. g \circ f$,

$\mathbf{A}(A, R-) : \mathbf{A} \rightarrow \mathbf{Set}$,

$\mathbf{A}(A, R-)_0 := \lambda D. \mathbf{A}(A, RD)$,

$\mathbf{A}(A, R-)_1 := \lambda g. \lambda h. Rg \circ h$,

$\alpha : \mathbf{B}(C, -) \rightarrow \mathbf{A}(A, R-)$,

$(\eta \mapsto \alpha_0) := \lambda \eta. \lambda D. \lambda f. Rf \circ \eta$,

$(\alpha \mapsto \eta) := \lambda \alpha. \alpha C(\text{id}_C)$,

or:

$\alpha_0 := \lambda D. \lambda f. Rf \circ \eta$,

$\eta := \alpha C(\text{id}_C)$.