

Cálculo 2 - 2025.1

Aulas 46 e 47: EDOs exatas

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<http://anggtwu.net/2025.1-C2.html>

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Método e exemplo

$$\begin{aligned}
 [E_5] &= \begin{pmatrix} dz = z_x dx + z_y dy = 0 \\ \frac{d}{dx} z = z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix} & [E_5][S_1] &= \begin{pmatrix} d(x^2 y^3) = (2xy^3)dx + (3x^2 y^2)dy = 0 \\ \frac{d}{dx}(x^2 y^3) = (2xy^3) + (3x^2 y^2) \frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix} \\
 [E_3] &= \begin{pmatrix} z_x dx + z_y dy = 0 \\ z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix} & [E_3][S_1] &= \begin{pmatrix} (2xy^3)dx + (3x^2 y^2)dy = 0 \\ (2xy^3) + (3x^2 y^2) \frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix} \\
 [E_2] &= \begin{pmatrix} z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix} & [E_2][S_1] &= \begin{pmatrix} (2xy^3) + (3x^2 y^2) \frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix} \\
 [S_1] &= \begin{bmatrix} z := (x^2 y^3) \\ z_x := (2xy^3) \\ z_y := (3x^2 y^2) \end{bmatrix} & [E_5][S_2] &= \begin{pmatrix} d(x^2 + y^2) = (2x)dx + (2y)dy = 0 \\ \frac{d}{dx}(x^2 + y^2) = (2x) + (2y) \frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix} \\
 [S_2] &= \begin{bmatrix} z := (x^2 + y^2) \\ z_x := (2x) \\ z_y := (2y) \end{bmatrix} & [E_3][S_2] &= \begin{pmatrix} (2x)dx + (2y)dy = 0 \\ (2x) + (2y) \frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix} \\
 & & [E_2][S_2] &= \begin{pmatrix} (2x) + (2y) \frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix}
 \end{aligned}$$

Uma questão da P2 de 2019.2

4) Sejam $(***)$ e $(****)$ estas EDOs:

$$\begin{aligned} 2xy^3 dx + 3x^2y^2 dy &= 0 & (***) \\ 2x^2y^3 dx + 3x^3y^2 dy &= 0 & (****) \end{aligned}$$

- a) **(0.5 pts)** Mostre que $(***)$ é exata.
- b) **(0.5 pts)** Encontre a solução geral de $(***)$.
- c) **(1.0 pts)** Teste a sua solução geral da $(***)$.
- d) **(0.5 pts)** Mostre que a solução geral da EDO $(***)$ também é solução da $(****)$.
- e) **(0.5 pts)** Mostre que $(****)$ não é exata.
- f) **(0.5 pts)** Mostre que o fator integrante obtido por

$$\begin{aligned} p(x) &= (M_y - N_x)/N, \\ \mu(x) &= e^{\int p(x) dx} \end{aligned}$$

transforma $(****)$ em $(***)$.

Versão original:

<http://angg.twu.net/LATEX/2019-2-C2-P2.pdf>

```
(%i1) load("2025-1-exact.mac")$
```

```
(%i2) ee : Exact_from0()$
```

```
(%i3) ee@@E5();
```

```
(%o3)
```

$$\begin{pmatrix} dz & = & z_x dx + z_y dy & = & 0 \\ \frac{d}{dx} z & = & z_x + z_y \left(\frac{d}{dx} y \right) & = & 0 \\ z & = & C \end{pmatrix}$$

```
(%i4) ee : Exact_from1(x^2+y^2)$
```

```
(%i5) ee@@E5();
```

```
(%o5)
```

$$\begin{pmatrix} d(y^2 + x^2) & = & 2x dx + 2y dy & = & 0 \\ \frac{d}{dx}(y^2 + x^2) & = & 2x + 2y \left(\frac{d}{dx} y \right) & = & 0 \\ y^2 + x^2 & = & C \end{pmatrix}$$

```
(%i6) ee : Exact_from2(2*x, 2*y)$
```

```
(%i7) ee@@E5();
```

```
(%o7)
```

$$\begin{pmatrix} dz & = & 2x dx + 2y dy & = & 0 \\ \frac{d}{dx} z & = & 2x + 2y \left(\frac{d}{dx} y \right) & = & 0 \\ z & = & C \end{pmatrix}$$

```
(%i8) ee@@ode();
```

```
(%o8)
```

$$2y \left(\frac{d}{dx} y \right) + 2x = 0$$

```
(%i9) ee@@odesolve();
```

```
(%o9)
```

$$-\left(\frac{y^2}{2}\right) = \frac{x^2}{2} + \%c$$

```
(%i10) sols : ee@@odesols();
```

```
(%o10)
```

$$\left[y = -\sqrt{-x^2 - 2\%c}, y = \sqrt{-x^2 - 2\%c} \right]$$

```
(%i11) sol : sols[1];
```

```
(%o11)
```

$$y = -\sqrt{-x^2 - 2\%c}$$

```
(%i12) test1 : ee@@odesubst(sol);
```

```
(%o12)
```

$$2x - 2\sqrt{-x^2 - 2\%c} \left(\frac{d}{dx} \left(-\sqrt{-x^2 - 2\%c} \right) \right) = 0$$

```
(%i13) test2 : ev(test1, diff);
```

```
(%o13)
```

$$0 = 0$$

```
(%i1) load("2025-1-exact.mac")$
(%i2) ee : Exact_from1(x^2*y^3);
(%o2)
Exact ( $x^2 y^3, 2 x y^3, 3 x^2 y^2$ )

(%i3) ee@@caixinhas_3();
(%o3)

$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{bmatrix}$$


(%i4) ee@@caixinhas_2();
(%o4)

$$\begin{bmatrix} \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{bmatrix}$$


(%i5) ee : Exact_from1(x^2*y^2);
(%o5)
Exact ( $y^2 + x^2, 2 x, 2 y$ )

(%i6) ee@@caixinhas_3();
(%o6)

$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{bmatrix}$$


(%i7) ee@@caixinhas_2();
(%o7)

$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{bmatrix}$$

```

```
(%i1) [maxxp,maxyp] : [2,2];
(%o1)
```

$$[2,2]$$

```
(%i2) "1"$ ee : Exact_from3("?", 2*x+3, 2*y-2);
(%o3)
```

$$\text{Exact}(\text{?}, 2x+3, 2y-2)$$

```
(%i4) ee@@caixinhas_2();
(%o4)
```

$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix} \end{bmatrix}$$

```
(%i5) ee@@effsc();
```

```
(%o5) 
$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} ? & 0 & 0 \\ ? & 0 & 0 \\ ? & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ ? & ? & ? \end{pmatrix} \end{bmatrix}$$

```

```
(%i6) "2"$ ee : Exact_from3("?", 2*x+4*y, 2*x-2*y);
(%o7)
```

$$\text{Exact}(\text{?}, 4y+2x, 2x-2y)$$

```
(%i8) ee@@caixinhas_2();
(%o8)
```

$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ -2 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix} \end{bmatrix}$$

```
(%i9) ee@@effsc();
```

```
(%o9) 
$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} ? & 0 & 0 \\ ? & 4 & 0 \\ ? & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ ? & ? & ? \end{pmatrix} \end{bmatrix}$$

```

```
(%i10) [maxxp,maxyp] : [3,3];
(%o10)
```

$$[3,3]$$

```
(%i11) "3"$ ee : Exact_from3("?", 3*x^2 - 2*x*y + 2, 6*y^2 - x^2 + 3);
(%o12)
```

$$\text{Exact}(\text{?}, -(2xy) + 3x^2 + 2, 6y^2 - x^2 + 3)$$

```
(%i13) ee@@caixinhas_2();
(%o13)
```

$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 2 & 0 & 3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & -1 & 0 \end{pmatrix} \end{bmatrix}$$

```
(%i14) ee@@effsc();
```

```
(%o14) 
$$\begin{bmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} ? & 0 & 0 & 0 \\ ? & 0 & 0 & 0 \\ ? & 0 & -1 & 0 \\ ? & 2 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ ? & ? & ? & ? \end{pmatrix} \end{bmatrix}$$

```