

Cálculo 2 - 2025.1

Aula 30: séries de Taylor

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<http://anggtwu.net/2025.1-C2.html>

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Vídeo do Mathologer – assista do 4:00 ao 12:21:

<http://www.youtube.com/watch?v=p-LbzWmm2zk>

Quadros:

2hQ32 (2023.2) Frações parciais
 2hQ70 (2023.2) Revisão de variáveis complexas
 2hQ75 (2023.2) EDOs exatas
 2hQ89 (2023.2) Séries de Taylor
 2hT234 Código em Maxima

```
(%i1) load("~/MAXIMA/2025-1-taylor.mac")$
(%i2) gradef(f(x), f_x(x))$
(%i3) gradef(f_x(x), f_xx(x))$
(%i4) gradef(f_xx(x), f_xxx(x))$
(%i5) gradef(f_xxx(x), f_xxxx(x))$
(%i6) gradef(f_xxxx(x), f_xxxxx(x))$
(%i7) myt : Taylor(f(x),x,0,4)$
(%i8) myt@@abcd(x);
(%o8) 
$$\begin{pmatrix} f(x) \\ f_x(x) \\ f_{xx}(x) \\ f_{xxx}(x) \\ f_{xxxx}(x) \end{pmatrix}, \begin{pmatrix} f(0) \\ f_x(0) \\ f_{xx}(0) \\ f_{xxx}(0) \\ f_{xxxx}(0) \end{pmatrix}, \begin{pmatrix} f(0) \\ f_x(0) \\ \frac{f_{xx}(0)}{2} \\ \frac{f_{xxx}(0)}{6} \\ \frac{f_{xxxx}(0)}{24} \end{pmatrix}, \begin{pmatrix} f(0) \\ f_x(0)x \\ \frac{f_{xx}(0)x^2}{2} \\ \frac{f_{xxx}(0)x^3}{6} \\ \frac{f_{xxxx}(0)x^4}{24} \end{pmatrix}$$

(%i9) myt@@s(x);
(%o9) 
$$\frac{f_{xxxx}(0)x^4}{24} + \frac{f_{xxx}(0)x^3}{6} + \frac{f_{xx}(0)x^2}{2} + f_x(0)x + f(0)$$

(%i10) myt@@s(a);
(%o10) 
$$\frac{f_{xxxx}(0)a^4}{24} + \frac{f_{xxx}(0)a^3}{6} + \frac{f_{xx}(0)a^2}{2} + f_x(0)a + f(0)$$

(%i11) myt@@s(1/10);
(%o11) 
$$\frac{f_{xxxx}(0)}{240000} + \frac{f_{xxx}(0)}{6000} + \frac{f_{xx}(0)}{200} + \frac{f_x(0)}{10} + f(0)$$

(%i12)
```

```

(%i1) load("-/MAXIMA/2025-1-taylor.mac")$
(%i2) f3(x) := a + b*x + c*x^2 + d*x^3$
(%i3) myt : Taylor(f3(x),x,0,4)$
(%i4) myt@@abcd(x);
(%o4)

$$\begin{bmatrix} dx^3 + cx^2 + bx + a \\ 3dx^2 + 2cx + b \\ 6dx + 2c \\ 6d \\ 0 \end{bmatrix}, \begin{bmatrix} a \\ b \\ 2c \\ 6d \\ 0 \end{bmatrix}, \begin{bmatrix} a \\ b \\ c \\ d \\ 0 \end{bmatrix}, \begin{bmatrix} a \\ bx \\ cx^2 \\ dx^3 \\ 0 \end{bmatrix}$$

(%i5) myt@@s(x);
(%o5)

$$dx^3 + cx^2 + bx + a$$

(%i6) f4(x) := a + b*x + c*x^2 + d*x^3 + e*x^4$
(%i7) myt : Taylor(f4(x),x,0,4)$
(%i8) myt@@abcd(x);
(%o8)

$$\begin{bmatrix} ex^4 + dx^3 + cx^2 + bx + a \\ 4ex^3 + 3dx^2 + 2cx + b \\ 12ex^2 + 6dx + 2c \\ 24ex + 6d \\ 24e \end{bmatrix}, \begin{bmatrix} a \\ b \\ 2c \\ 6d \\ 24e \end{bmatrix}, \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}, \begin{bmatrix} a \\ bx \\ cx^2 \\ dx^3 \\ ex^4 \end{bmatrix}$$

(%i9) myt@@s(x);
(%o9)

$$ex^4 + dx^3 + cx^2 + bx + a$$

(%i10) f5(x) := a + b*x + c*x^2 + d*x^3 + e*x^4 + f*x^5$
(%i11) myt : Taylor(f5(x),x,0,4)$
(%i12) myt@@abcd(x);
(%o12)

$$\begin{bmatrix} fx^5 + ex^4 + dx^3 + cx^2 + bx + a \\ 5fx^4 + 4ex^3 + 3dx^2 + 2cx + b \\ 20fx^3 + 12ex^2 + 6dx + 2c \\ 60fx^2 + 24ex + 6d \\ 120fx + 24e \end{bmatrix}, \begin{bmatrix} a \\ b \\ 2c \\ 6d \\ 24e \end{bmatrix}, \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}, \begin{bmatrix} a \\ bx \\ cx^2 \\ dx^3 \\ ex^4 \end{bmatrix}$$

(%i13) myt@@s(x);
(%o13)

$$ex^4 + dx^3 + cx^2 + bx + a$$


```

```
(%i1) load("-/MAXIMA/2025-1-taylor.mac")$
(%i2) myte : Taylor(exp( th) , th,0,5)$
(%i3) myte2 : Taylor(exp( 2*th) , th,0,5)$
(%i4) mytei : Taylor(exp(xi*th) , th,0,5)$
(%i5) mytc : Taylor(cos(th) , th,0,5)$
(%i6) myts : Taylor( sin(th), th,0,5)$
(%i7) mytis : Taylor( %i*sin(th), th,0,5)$
(%i8) mytcis : Taylor(cos(th)*%i*sin(th), th,0,5)$
(%i9) myte @@abcd(th);
```

$$\begin{pmatrix} e^{\theta} \\ e^{\theta} \\ e^{\theta} \\ e^{\theta} \\ e^{\theta} \\ e^{\theta} \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ \theta \\ \frac{\theta^2}{2!} \\ \frac{\theta^3}{3!} \\ \frac{\theta^4}{4!} \\ \frac{\theta^5}{5!} \end{pmatrix}$$

```
(%i10) myte2 @@abcd(th);
(%i10)
```

$$\begin{pmatrix} e^{2\theta} \\ 2e^{2\theta} \\ 4e^{2\theta} \\ 8e^{2\theta} \\ 16e^{2\theta} \\ 32e^{2\theta} \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \\ 8 \\ 16 \\ 32 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2\theta \\ 2\theta^2 \\ 4\theta^3 \\ 8\theta^4 \\ 16\theta^5 \end{pmatrix}$$

```
(%i11) mytei @@abcd(th);
(%i11)
```

$$\begin{pmatrix} e^{i\theta} \\ i e^{i\theta} \\ -e^{i\theta} \\ -i e^{i\theta} \\ e^{i\theta} \\ i e^{i\theta} \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -i \\ 1 \\ i \end{pmatrix}, \begin{pmatrix} 1 \\ i \\ \frac{1}{2} \\ \frac{1}{6} \\ \frac{1}{24} \\ \frac{1}{120} \end{pmatrix}, \begin{pmatrix} i\theta \\ -\frac{\theta^2}{2} \\ \frac{i\theta^3}{6} \\ -\frac{\theta^4}{24} \\ \frac{i\theta^5}{120} \end{pmatrix}$$

```
(%i12) mytc @@abcd(th);
(%i12)
```

$$\begin{pmatrix} \cos \theta \\ -\sin \theta \\ -\cos \theta \\ \sin \theta \\ \cos \theta \\ -\sin \theta \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{24} \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -\frac{\theta^2}{2} \\ 0 \\ \frac{\theta^4}{24} \\ 0 \end{pmatrix}$$

```
(%i13) myts @@abcd(th);
(%i13)
```

$$\begin{pmatrix} \sin \theta \\ \cos \theta \\ -\sin \theta \\ -\cos \theta \\ \sin \theta \\ \cos \theta \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{1}{6} \\ 0 \\ \frac{1}{120} \end{pmatrix}, \begin{pmatrix} 0 \\ \theta \\ 0 \\ -\frac{\theta^3}{6} \\ 0 \\ \frac{\theta^5}{120} \end{pmatrix}$$

```
(%i14) mytis @@abcd(th);
(%i14)
```

$$\begin{pmatrix} i \sin \theta \\ i \cos \theta \\ -(i \sin \theta) \\ -(i \cos \theta) \\ i \sin \theta \\ i \cos \theta \end{pmatrix}, \begin{pmatrix} 0 \\ i \\ 0 \\ -i \\ 0 \\ i \end{pmatrix}, \begin{pmatrix} 0 \\ i \\ 0 \\ -\frac{1}{6} \\ 0 \\ \frac{1}{120} \end{pmatrix}, \begin{pmatrix} 0 \\ i\theta \\ 0 \\ -\frac{i\theta^3}{6} \\ 0 \\ \frac{i\theta^5}{120} \end{pmatrix}$$

```
(%i15) mytcis @@abcd(th);
(%i15)
```

$$\begin{pmatrix} i \sin \theta + \cos \theta \\ i \cos \theta - \sin \theta \\ -(i \sin \theta) - \cos \theta \\ \sin \theta - i \cos \theta \\ i \sin \theta + \cos \theta \\ i \cos \theta - \sin \theta \end{pmatrix}, \begin{pmatrix} 1 \\ i \\ -1 \\ 1 \\ i \end{pmatrix}, \begin{pmatrix} 1 \\ i \\ -\frac{1}{2} \\ \frac{1}{6} \\ \frac{1}{24} \\ \frac{1}{120} \end{pmatrix}, \begin{pmatrix} 1 \\ i\theta \\ -\frac{\theta^2}{2} \\ 0 \\ \frac{i\theta^3}{6} \\ \frac{\theta^4}{24} \\ \frac{i\theta^5}{120} \end{pmatrix}$$

```
(%i16) mytei @@s(th);
(%i16)
```

$$\frac{i\theta^5}{120} + \frac{\theta^4}{24} - \frac{i\theta^3}{6} - \frac{\theta^2}{2} + i\theta + 1$$

```
(%i17) mytcis @@s(th);
(%i17)
```

$$\frac{i\theta^5}{120} + \frac{\theta^4}{24} - \frac{i\theta^3}{6} - \frac{\theta^2}{2} + i\theta + 1$$