

On the Shapes of Proofs in Basic Calculus

(Preliminary version)

<http://anggtwu.net/math-b.html#2025-sbc>

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CWM

From CWM, p.84:

$$\begin{aligned}
 \varphi\theta g &= G\varepsilon_a \circ GFg \circ \eta_x \\
 &= G\varepsilon_a \circ \eta_{Ga} \circ g \\
 &= g
 \end{aligned}$$

$$\begin{aligned}
 \varphi\theta g &= \varphi(\varepsilon_a \circ Fg) && \text{by } def_\theta \\
 &= G(\varepsilon_a \circ Fg) \circ \eta_x && \text{by } def_\varphi. \\
 &= G\varepsilon_a \circ GFg \circ \eta_x && \text{by } respcomp_G \\
 &= G\varepsilon_a \circ \eta_{Ga} \circ g && \text{by } sqcond_\eta \\
 &= id_{Ga} \circ g && \text{by } G\varepsilon \circ \eta G = 1 \\
 &= g
 \end{aligned}$$

O aqui e agora não existe mais

Tic Tac Toe

×	○	
×	○	
X	O	

$$\begin{array}{lcl}
F(x) & = & (6x^3)(7x^4) \\
F'(x) & = & (F(x))' \\
& = & ((6x^3)(7x^4))' \\
& = & \frac{d}{dx}((6x^3)(7x^4)) \\
& = & (6x^3)\frac{d}{dx}(7x^4) + (7x^4)\frac{d}{dx}(6x^3) \\
& = & (6x^3)\frac{d}{dx}(7x^4) + (7x^4) \cdot 6\frac{d}{dx}x^3 \\
& = & (6x^3)\frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
& = & (6x^3)\frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
& = & (6x^3) \cdot 7 \cdot \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
\stackrel{(10)}{=} & (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) & RPowx \\
& = & (6x^3)(28x^3) + (7x^4)(18x^2) \\
& = & (6x^3)(28x^3) + 126x^6 \\
& = & 168x^6 + 126x^6 \\
& = & 294x^6
\end{array}
\qquad
\begin{array}{lcl}
F(x) & =_1 & (6x^3)(7x^4) \\
F'(x) & =_2 & ((F(x))_{3L})'_{2R} \\
& =_3 & ((6x^3)(7x^4)_{3R})'_{4L} \\
& =_4 & \frac{d}{dx}((6x^3)(7x^4))_{4L} \\
& =_5 & (6x^3)\frac{d}{dx}(7x^4) + (7x^4)\frac{d}{dx}(6x^3)_{6L} \quad 5R \\
& =_6 & (6x^3)\frac{d}{dx}(7x^4) + (7x^4) \cdot 6\frac{d}{dx}x^3_{7L} \quad 6R \\
& =_7 & (6x^3)\frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2_{7R} \quad 8L \\
& =_8 & (6x^3)\frac{d}{dx}(7x^4)_{9L} + (7x^4)(18x^2)_{8R} \\
& =_9 & (6x^3) \cdot 7\frac{d}{dx}x^4_{10L} \quad 9R + (7x^4)(18x^2) \\
& =_{10} & (6x^3) \cdot 7 \cdot 4x^3_{10R} \quad 11L + (7x^4)(18x^2) \\
& =_{11} & (6x^3)(28x^3)_{11R} + (7x^4)(18x^2)_{12L} \\
& =_{12} & (6x^3)(28x^3)_{13L} + 126x^6_{12R} \\
& =_{13} & 168x^6_{13R} + 126x^6_{14L} \\
& =_{14} & 294x^6_{14R}
\end{array}$$

$$\begin{aligned}
 F(x) &= (6x^3)(7x^4) \\
 F'(x) &= (F(x))' \\
 &= ((6x^3)(7x^4))' \\
 &= \frac{d}{dx}((6x^3)(7x^4))
 \end{aligned}$$

$$\begin{aligned}
 &\stackrel{(5)}{=} (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
 &= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
 &= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
 &= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
 &= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
 &= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
 &= (6x^3)(28x^3) + (7x^4)(18x^2) \\
 &= (6x^3)(28x^3) + 126x^6 \\
 &= 168x^6 + 126x^6 \\
 &= 294x^6
 \end{aligned}$$

$$\text{Por } \underbrace{\overbrace{[RProd]}^{f(x) := 6x^3} \left[g(x) := 7x^4 \right]}_{\left(\frac{d}{dx}(f(x)g(x)) = f(x) \frac{d}{dx}g(x) + g(x) \frac{d}{dx}f(x) \right)} \left(\frac{d}{dx}((6x^3)(7x^4)) = (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \right)$$

$$\begin{aligned}
 F(x) &= (6x^3)(7x^4) \\
 F'(x) &= (F(x))' \\
 &= ((6x^3)(7x^4))' \\
 &= \frac{d}{dx}((6x^3)(7x^4)) \\
 &= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3)
 \end{aligned}$$

$$\begin{aligned}
 \stackrel{(6)}{=} & (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
 &= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
 &= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
 &= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
 &= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
 &= (6x^3)(28x^3) + (7x^4)(18x^2) \\
 &= (6x^3)(28x^3) + 126x^6 \\
 &= 168x^6 + 126x^6 \\
 &= 294x^6
 \end{aligned}$$

$$\text{Por } \underbrace{\overbrace{[\text{RMC}]} \left[\begin{array}{l} c := 6 \\ f(x) := x^3 \end{array} \right]}_{\left(\frac{d}{dx}(cf(x)) = c \frac{d}{dx}f(x) \right)} \\
 \underbrace{\left(\frac{d}{dx}(6x^3) = 6 \frac{d}{dx}x^3 \right)}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&\stackrel{(7)}{=} (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \quad \text{Por } \underbrace{\text{[RPot]}}_{\left(\frac{d}{dx}x^n = nx^{n-1}\right)} [n := 3] \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&\stackrel{(9)}{=} (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \quad \text{Por } \underbrace{\text{[RMC]}}_{\left(\frac{d}{dx}(cf(x)) = c \frac{d}{dx}f(x) \right)} \left[\begin{array}{l} c := 7 \\ f(x) := x^4 \end{array} \right] \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&\stackrel{(10)}{=} (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \quad \text{Por } \underbrace{\text{[RPot]}}_{\left(\frac{d}{dx}x^n = nx^{n-1}\right)} [n := 4] \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6 \quad \underbrace{\left(\frac{d}{dx}x^4 = nx^{4-1}\right)}
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &\stackrel{(2)}{=} (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&\stackrel{(3)}{=} ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
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\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&\stackrel{(4)}{=} \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
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$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&\stackrel{(5)}{=} (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \quad RProd \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
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F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&\stackrel{(6)}{=} (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \quad RMC \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
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&= 168x^6 + 126x^6 \\
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\end{aligned}$$

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F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot \frac{d}{dx}x^3 \\
&\stackrel{(7)}{=} (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \quad RPower \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
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$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
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&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&\stackrel{(8)}{=} (6x^3) \frac{d}{dx}(7x^4) + (7x^4) (18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
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\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&\stackrel{(9)}{=} (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \quad RMC \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&\stackrel{(10)}{=} (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \quad RPowx \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot \boxed{7 \cdot 4x^3} + (7x^4)(18x^2) \\
&\stackrel{(11)}{=} (6x^3) \boxed{(28x^3)} + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&\stackrel{(12)}{=} (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&\stackrel{(13)}{=} 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}$$

$$\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \frac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \frac{d}{dx}(6x^3) \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \frac{d}{dx}x^3 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 \\
&= (6x^3) \frac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \frac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&\stackrel{(14)}{=} 294x^6
\end{aligned}$$