

Notes on Emily Riehl's book ("Categories in Context")

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<https://anggtwu.net/math-b.html>

Horizontal and vertical compositions

From [Riehl, p.46]...

In my notation:

an outer $[\cdot]$ stresses that the result is a NT, and

an outer $(\cdot)$ stresses that the result is a functor.

$$\begin{array}{c}
 \begin{array}{ccccc}
 & F & & J & \\
 & \curvearrowright & & \curvearrowright & \\
 \mathbf{C} & \xrightarrow{G} & \mathbf{D} & \xrightarrow{K} & \mathbf{E} \\
 & \curvearrowleft & & \curvearrowleft & \\
 & H & & L &
 \end{array} \\
 \\
 \begin{array}{c}
 \begin{array}{ccccc}
 & F & & J & \\
 & \curvearrowright & & \curvearrowright & \\
 \mathbf{C} & \xrightarrow{\Downarrow \beta \cdot \alpha} & \mathbf{D} & \xrightarrow{\Downarrow \delta \cdot \gamma} & \mathbf{E} \\
 & \curvearrowleft & & \curvearrowleft & \\
 & H & & L &
 \end{array} = \begin{array}{ccc}
 & JF & \\
 & \curvearrowright & \\
 \mathbf{C} & \xrightarrow{KG} & \mathbf{E} \\
 & \curvearrowleft & \\
 & LH & \\
 & F, J &
 \end{array} \quad (\delta \cdot \gamma) * (\beta \cdot \alpha) = (\delta * \beta) \cdot (\gamma * \alpha) \\
 \\
 \begin{array}{c}
 \begin{array}{ccccc}
 & F & & J & \\
 & \curvearrowright & & \curvearrowright & \\
 \mathbf{C} & \xrightarrow{\Downarrow [\alpha \atop \beta]} & \mathbf{D} & \xrightarrow{\Downarrow [\gamma \atop \delta]} & \mathbf{E} \\
 & \curvearrowleft & & \curvearrowleft & \\
 & H & & L &
 \end{array} = \begin{array}{ccc}
 & \Downarrow \alpha, \gamma & \\
 & \curvearrowright & \\
 \mathbf{C} & \xrightarrow{G, K} & \mathbf{E} \\
 & \curvearrowleft & \\
 & \Downarrow \beta, \delta & \\
 & H, L &
 \end{array} \quad \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \begin{bmatrix} \gamma \\ \delta \end{bmatrix} = \begin{bmatrix} \alpha, \gamma \\ \beta, \delta \end{bmatrix} : (F, J) \xrightarrow{\bullet} (H, L)
 \end{array}
 \end{array}$$

$$\mathbf{C} \xrightarrow{F} \mathbf{D} \begin{array}{c} \xrightarrow{H} \\ \Downarrow \beta \\ \xrightarrow{K} \end{array} \mathbf{E} \xrightarrow{L} \mathbf{F}$$

$$L\beta F : LHF \Rightarrow LKF$$

$$[F, \beta, L] : (F, H, L) \xrightarrow{\bullet} (F, K, L)$$

$$\begin{array}{ccccc} & F & & J & \\ & \Downarrow \alpha & & \Downarrow \gamma & \\ \mathbf{C} & \xrightarrow{G} & \mathbf{D} & \xrightarrow{K} & \mathbf{E} \\ & \Downarrow \beta & & \Downarrow \delta & \\ & H & & L & \end{array}$$

$$\begin{array}{ccccc} & F & & J & \\ & \Downarrow \beta \cdot \alpha & & \Downarrow \delta \cdot \gamma & \\ \mathbf{C} & \xrightarrow{H} & \mathbf{D} & \xrightarrow{L} & \mathbf{E} \\ & & & & \end{array} = \begin{array}{ccc} & JF & \\ & \Downarrow \gamma * \alpha & \\ \mathbf{C} & \xrightarrow{KG} & \mathbf{E} \\ & \Downarrow \delta * \beta & \\ & LH & \\ & (F, J) & \end{array} \quad (\delta \cdot \gamma) * (\beta \cdot \alpha) = (\delta * \beta) \cdot (\gamma * \alpha)$$

$$\begin{array}{ccccc} & F & & J & \\ & \Downarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} & & \Downarrow \begin{bmatrix} \gamma \\ \delta \end{bmatrix} & \\ \mathbf{C} & \xrightarrow{H} & \mathbf{D} & \xrightarrow{L} & \mathbf{E} \\ & & & & \end{array} = \begin{array}{ccc} & \Downarrow \begin{bmatrix} \alpha, \gamma \\ \beta, \delta \end{bmatrix} & \\ \mathbf{C} & \xrightarrow{(G, K)} & \mathbf{E} \\ & \Downarrow \begin{bmatrix} \alpha, \gamma \\ \beta, \delta \end{bmatrix} & \\ & (H, L) & \end{array} \quad \left[\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \right] = \left[\begin{bmatrix} \alpha, \gamma \\ \beta, \delta \end{bmatrix} \right]$$

$$\begin{array}{ccccc} & F & & J & \\ & \Downarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} & & \Downarrow \begin{bmatrix} \gamma \\ \delta \end{bmatrix} & \\ \mathbf{C} & \xrightarrow{H} & \mathbf{D} & \xrightarrow{L} & \mathbf{E} \\ & & & & \end{array} = \begin{array}{ccc} & F, J & \\ & \Downarrow \alpha, \gamma & \\ \mathbf{C} & \xrightarrow{G, K} & \mathbf{E} \\ & \Downarrow \beta, \delta & \\ & H, L & \end{array} \quad \left[\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \right] = \left[\begin{bmatrix} \alpha, \gamma \\ \beta, \delta \end{bmatrix} \right]$$

Horizontal composition

